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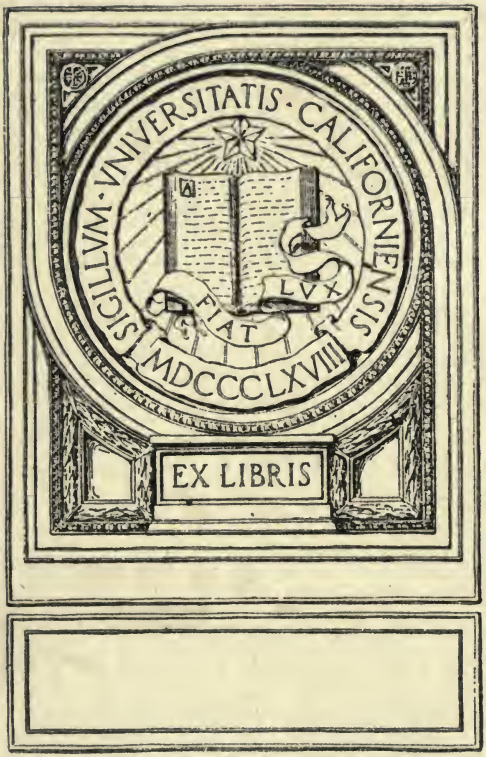
THE MILITARY MAP

FRENCH SCHOOL OF WAR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THE MILITARY MAP

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THE MILITARY MAP

ELEMENTS OF

MODERN TOPOGRAPHY

(FRENCH SCHOOL OF WAR)

UNIV. OF CALIFORNIA

MACMILLAN AND CO., LIMITED
ST. MARTIN'S STREET, LONDON
1916

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PREFACE.

THE military map of to-day is constructed on a solid basis. The geometrical principles involved may, however, be adequately explained without the use of any more formidable mathematical apparatus than the simplest of simple equations.

Such elementary treatment will, perhaps, appear beneath the dignity of the subject. But that does not matter if this little book leads to a study of the exhaustive treatises on which it is founded. For the young topographer will thus, by easy stages, have been brought in view of some quite pretty problems that give practical results of ever-increasing interest and value.

The work followed in the main is the *Cours de Topographie Élémentaire* of Major Emile Espérandieu, with the consent and kind co-operation of the author. The excellent *Manuel à l'usage des Sociétés de Préparation militaire* for infantry by Lt.-Colonel Hatton, and the admirable *Topographie Pratique* of Captain E. de Larminat have also been laid freely under contribution by courteous permission of Monsieur Charles-Lavauzelle, official military publisher, of Paris and Limoges.

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No apology is offered for the retention of a few French terms; since here, as elsewhere in the world of science and art, France has led the way. But where an appropriate English equivalent could be found it has always been used.

Dr. Robert Lachlan (author of *Modern Pure Geometry*) has written an introduction which throws fresh light on the initial process of map-making; and he has done much to make what follows complete, concise, and clear.

G. M.

March, 1916.

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INTRODUCTION.

A MAP is a picture drawn on a flat piece of paper of a portion of the earth's surface and gives a similar view of the earth to that presented to an observer in the car of a balloon floating in the air. As a balloon rises an observer would gradually lose sight of some of the details, such as hedges, streams, and even roads; at first he would see the surface of the earth in relief, and he would be able to distinguish all the undulations of the ground; but as he rose higher the country would tend to look flat, and at a height of only a few thousand feet an inexperienced observer would find difficulty in distinguishing the valleys from the hills. In certain atmospheric conditions the surface of the earth as seen from a balloon looks quite flat at moderate heights—even as low as 2000 feet, provided none of the hills rise above 1000 feet. In such circumstances the view seen from a balloon exactly resembles the picture which it is the aim of the map-maker to produce.

The object of this book is to discuss the principal methods used in map-making; to explain the difficulties which arise, and the means by which they are surmounted; also to indicate the limitations of the different kinds of maps, and illustrate the descriptive

geometrical manner in which variations of altitude are now represented.

In making a map the first point to consider is the purpose for which it is required. For use in connection with Local Government such details as the boundaries of parishes and other districts would have to be inserted, and it is most desirable that the area enclosed by any boundary should be correctly represented. For military purposes parish and other local boundaries are of no importance, and may therefore be omitted, but roads, railways, canals, and rivers must be inserted with the utmost accuracy. It is, of course, also desirable to indicate the character of the country, so as to show as clearly as possible the extent of marshes and other open spaces, together with the position of all the hills and the relative steepness of their slopes.

To understand the nature of the difficulties which confront the map-maker it will be sufficient to consider a simple case. Suppose a farm to be situated in a flat part of the country. By taking a few measurements it would be easy to draw to scale a plan of each field, and then by fitting together the plans of the different fields it would be possible to obtain a plan of the whole farm. It might be thought that the same method could be used for obtaining a map of any extent of flat country. But, as soon as the extent of the country exceeded sixty miles in any direction, it would be found impossible to fit together the plans of the smaller areas into which the country had been divided for the purpose of

measurement. The reason why this is impossible is because the earth is round and not flat. This is the first difficulty, and various ways of surmounting it have been suggested.

Although there are flat stretches of country in many different parts of the world, the greater part of the land surface is by no means flat, but broken up into hilly ground and mountainous regions intersected by valleys. Thus a second difficulty is presented, but it is one which is more easily overcome. Suppose that a railway is constructed across one of the fields of a farm, for which a plan has been made; then, whether the track is laid on an embankment or in a cutting, the space occupied by it will be properly indicated on the plan by the space taken away from the field. For if the railway was afterwards abandoned, and if the farmer were to cultivate the ridge or the hollow it had occupied with the rest of the field, he would be able to grow just as much and no more than before the railway was made, because corn and all other plants grow vertically. Similarly, in the case of any field which is not in a horizontal plane no greater crop can be raised than that which could be raised on the horizontal space covered by the field. Consequently the plan of a farm in which every field is represented by the horizontal space which it covers gives a correct measure of the productive area of the farm. But in drawing a map every hill, whatever its size or shape, must necessarily be regarded as standing on a horizontal plane, and be represented proportionally

on the map by the space which it covers on that plane.

Now comes the question as to how hills can be shown so as to indicate clearly the steepness of their slopes. One method is suggested by the view from a balloon. When an experienced observer has risen to a height at which the surface of the earth looks flat, he recognises the hills and valleys by the differences of light and shade. For instance, if the sun is in the west the eastern side of a valley which runs due south will receive more light than the western side of the valley, but for a mountain ridge the opposite will be the case. Accordingly, hills are sometimes represented on maps by means of shading: every part of the map showing to what extent the corresponding part of the earth's surface is in shade when the sun is in the west. This method, however, does not give sufficiently good results, especially in the case of military maps. For such maps the "contour" method and the "hachure" method, both of which will be fully explained later, are the best to employ, although in some cases considerable advantage is gained by also using the method of shading.

THE EARTH.

The earth is a spherical body moving in space. It is the third known planet in order of distance from the sun. The motion of the earth is twofold. The centre of the earth travels in an elliptical, but nearly

circular, path or orbit round the sun in 365¼ days, always remaining in the same plane with the centre of the sun; and every 24 hours the earth turns about an imaginary axis passing through its centre

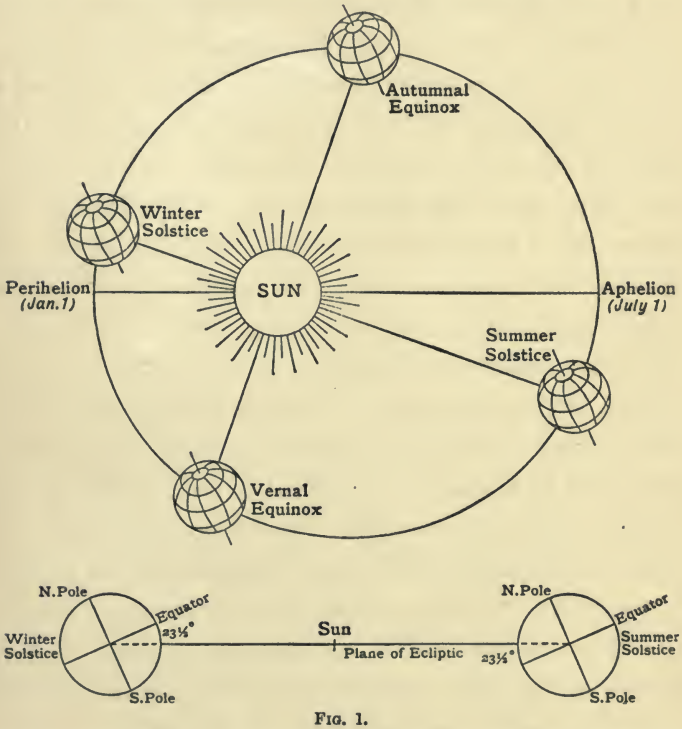

FIG. 1.

and inclined at a constant angle (about 23½ degrees) to the plane in which its centre moves.

The earth is not exactly a perfect sphere, but is what is called an oblate spheroid. If cut by a plane perpendicular to the axis about which it turns the section is a circle; but if cut by a plane containing the axis the section is an ellipse of which the greatest diameter is 7926 miles, and the smallest diameter

is 7900 miles—the latter coinciding with the axis of the earth, the former being perpendicular to it. Thus the earth is very nearly a perfect sphere, and for most practical purposes it may be treated as one.

As the earth is practically a sphere a few simple terms used in spherical trigonometry may be employed to express its geometrical lines. All plane sections of a sphere are circles. Plane sections passing through the centre of the sphere are called *great* circles; and plane sections which do not pass through the centre of the sphere are called *small* circles. All great circles are of the same size, because the diameter of any one is a diameter of the sphere.

By the *distance* between any two points on the earth is meant the length of the shortest path between them measured on the surface. This will be the length of the arc of the great circle which passes through the two points. The distance from any point on the earth to a curve traced on its surface is the length of the shortest path from the point to the curve measured on the surface, and so will be the arc of the great circle whose plane passes through the point and is perpendicular to the curve.

It is often convenient to measure the distance between two points on a sphere by the angle subtended at the centre of the sphere. Thus if O denotes the centre of the earth the distance between two points P and Q is measured by the angle POQ. If the number of degrees * in the angle POQ is multiplied

*In England angles are measured in degrees (°), minutes (′), and seconds (″). The right angle is divided into 90 equal parts called

by 69 the result gives the distance *PQ* expressed in miles; and if the number of grades in the angle *POQ* is multiplied by 100 the result gives the distance of *PQ* in kilometres.

The extremities of the earth's axis are called the North and South *poles*.

The great circle in which the plane through the centre that is perpendicular to the axis cuts the earth's surface is called the *equator*. The equator is equidistant from the two poles; and the plane of the equator divides the earth into two equal parts called the northern and southern hemispheres.

The plane passing through any point on the earth and also through the axis of the earth is called the

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degrees; the degree is divided into 60 equal parts called minutes; and the minute is divided into 60 equal parts called seconds.

In France, for all purposes connected with maps, angles are measured in grades (ᵍ), minutes (ᶜ), and seconds (ᶜᶜ). The right angle is divided into 100 equal parts called grades; the grade is divided into 100 equal parts called minutes; and the minute is divided into 100 equal parts called seconds.

To compare grades with degrees, or *vice versâ*:—Suppose the measure of an angle is D degrees or G grades.

Then $\frac{D}{90}=\frac{G}{100}$; and therefore $D=G-\frac{1}{10}G$;

that is, one tenth of the number of grades in an angle must be subtracted from the number to obtain the number of degrees.

Also, $G=D+\frac{1}{9}D$; that is, one ninth of the number of degrees in an angle must be added to the number to obtain the number of grades.

To convert an angle expressed in English measure to French measure, the best method is first to express the angle in degrees and decimals of a degree.

Thus 63° 27′ 44″ = 63·4622 degrees; one ninth of 63·4622 = 7·0514.

Therefore the size of the angle in French measure is

70·5136 grades, that is, 70ᵍ 51ᶜ 36ᶜᶜ.

meridian plane of the point; it cuts the earth in a great circle passing through the poles, and that portion of the great circle between the poles which contains the point is called the *meridian line*, or simply the *meridian* of the point.

The angle between the meridian plane of a point and a fixed meridian plane is called the *longitude* of the point. In English maps the fixed meridian

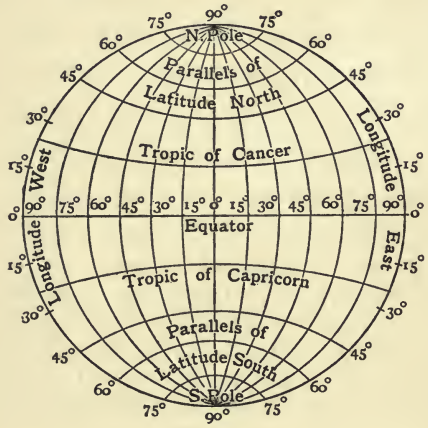

FIG. 2.

plane is the one which passes through Greenwich Observatory; in France the fixed meridian plane is that which passes through the Observatory of Paris.* Longitude is measured East or West of the fixed plane up to 180° or 200^g either way.

All meridian planes are perpendicular to the equator, and the angle between two meridian planes is the

*By the law of Feb. 15, 1911, Greenwich time was adopted in France, and quite recently the meridian of Greenwich has been taken as the meridian of origin.

same as the angle subtended at the centre of the earth by the arc of the equator cut off by these planes. Thus, if N is the north pole and if P and Q are any two points on the earth, the angle between the meridians NP and NQ is the same as the angle HOK, where H and K are the points in which the meridians NP and NQ cut the equator, and O is the centre of the earth.

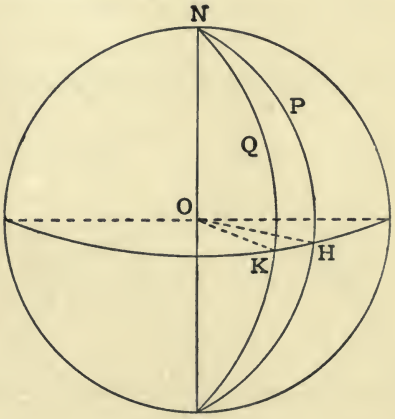

FIG. 3.

The angle which the vertical line at any point of the earth makes with the plane of the equator is called the *latitude* of the point. If the meridian NPS of a point P cuts the equator in H the angle subtended at the centre of the earth by the arc HP of the meridian is approximately * equal to the latitude

* Where it becomes necessary to take into account the spheroidal shape of the earth, it will be seen that the latter definition is not sufficiently accurate (Fig. 3A).

Let P be any point on the earth, and NPH the meridian of P cutting the equator in H. Draw the tangent PT, touching the meridian at

of P—if the earth were a perfect sphere the two angles would be exactly equal. Latitude is measured up to 90° or 100^g north or south from the equator.

The small circles in which planes parallel to the equator cut the earth's surface are called *parallels of latitude*, for the latitude of every point on such a circle is the same.

The latitude and longitude of a point on the earth are called the geographical *co-ordinates* * of the

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P, and draw PG perpendicular to PT, meeting the plane of the equator in G.

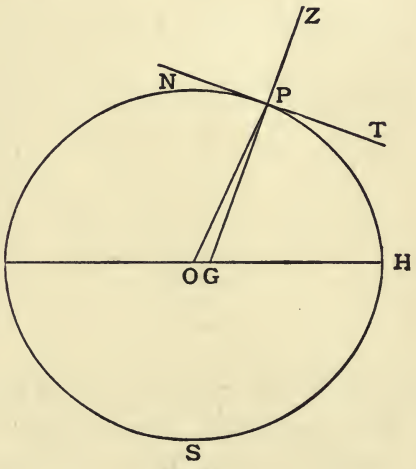

Fig. 3A.

The angle POH is called the *geocentric* latitude of P, and the angle PGH is called the *geographical* latitude of P.

If one were actually at the point P, PT would be the horizontal line running North and South, and PG would be the vertical line through P.

* As long ago as 1637 the French philosopher Descartes thus described the co-ordinates which bear his name:

Suppose that two fixed straight lines XOX', YOY' intersect in O. Through any point P let PN and PM be drawn parallel to XX' and

point. They enable one to ascertain and fix the exact position of a point, having regard to the true shape of the earth. They can be found at any place by astronomical observations, and by various methods are always very carefully determined for all the principal stations in the trigonometrical survey of a country.

The equator is a circle of which the circumference measures forty million metres or 24,900 miles. The distance measured along the equator between two meridians which differ by one degree of longitude is 111·11 kilometres or 69·17 miles. If the earth were a perfect sphere the distance measured along a

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YY'. Then when the position of *P* is known the lengths of *PN* and *PM* are also known; and when the lengths of *PN* and *PM* are given the position of *P* can be determined.

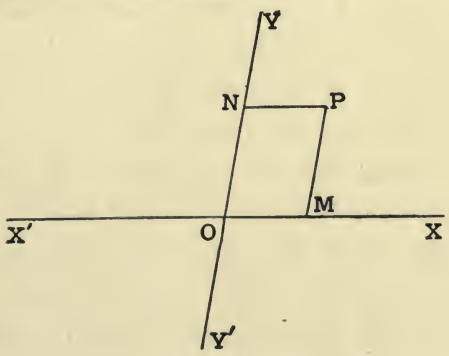


Fig. 4.

PM, usually denoted by *y*, is called the ordinate of *P*; and *PN*, or *OM* the part cut off by the ordinate, is called the abscissa of *P*, and is usually denoted by *x*.

XX' and *YY'* are called the *axes* of co-ordinates, and *O* is called the origin.

The *axes* are said to be rectangular or oblique according as they intersect at right angles or not.

meridian between any two parallels differing by one degree of latitude would be the same, that is, 69·17 miles; but owing to the spheroidal form of the earth the meridian length of one degree of latitude varies, increasing slowly from 68·70 miles at the equator to 69·41 miles at the pole.

The length of one degree of longitude measured along a parallel of latitude decreases as the latitude increases. At the equator the length of one degree of longitude is 69·17 miles; in latitude 60° the length of one degree of longitude is 34·67 miles; and at the pole the length is zero.

SURVEYING.

All maps are based upon actual measurements. The process of taking the necessary measurements is called surveying. The methods used in making a survey of a large tract of country at the present day are very elaborate, but the main principles on which the methods are founded are not difficult to understand.

In order to draw a plan for a flat field the measurements might be made with a tape-measure. If the field was triangular with straight sides, it would be sufficient to measure the length of each side. If the field was bounded by four or more straight sides, the plan of the field could be drawn after dividing it into triangles and then measuring the sides of each triangle.

This method of carrying out a survey by dividing the country into triangles is known as *triangulation*.

But, as it is very difficult to measure the length of a line by means of a tape at all accurately, and also because it is not always practicable to measure the distance between two points, those engaged in a survey chiefly use instruments for measuring angles.

Now a triangle can always be drawn when the length of one side is known, as well as the size of two of its angles.* For instance, if in the triangle ABC the length of AB is known, and if the size of the angles ABC and BAC is also known, the triangle can be drawn. Then the lengths of the other sides can be found by measuring them in the drawing. At the same time the lengths of these sides could be found by trigonometrical calculation.

Suppose, for the purpose of surveying a region which for the present is assumed to be in a horizontal plane, that A, B, C, D, ... are the vertices of the triangles into which the region has been divided. Let AB be measured and also the angles ABC and BAC: then the lengths of AC and BC can be determined. Then in the triangle DAC, since the length of AC has been determined, by measuring the angles CAD and ACD the lengths of the sides AD and CD can be determined. Proceeding in the same way the lengths of the sides of each of the triangles into which the region has been divided can be determined, and then a plan can be drawn.

Suppose now that the region to be surveyed is not a horizontal plane. It will usually be possible

* In any plane triangle the sum of the three angles is 180°; consequently, if any two angles are known the third is also known.

to select two points A and B which are in the same horizontal line, and to measure AB as the base line. Then let the region be divided into triangles, of which $C, D, E, \ldots$ are the vertices :—it will generally be advisable to select for these points the summit

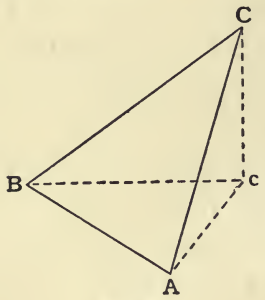


FIG. 5.

of the hills and other prominent points. Consider the triangle ABC: let c be the point in the horizontal plane containing AB which is vertically under C; then by means of an instrument called a theodolite the angle BAc, that is, the angle between the vertical plane containing AC and the vertical plane containing AB, can be measured; and similarly, the angle ABc can be measured. Consequently, since the length of AB is known, the sides Ac and Bc can be determined. Again, the angle cAC, that is, the angle of elevation of C as seen from A, can be measured with the same instrument, and then, in the triangle CAc, since the angle at c is a right angle and the length of Ac has been already found, the lengths of AC and Cc can be determined.

Next consider a point D which is visible from A and C, but is not visible from B. In the triangle

ACD the length of *AC* has already been found; the angles *CAD* and *ACD* can be measured, and so the lengths of the sides *AD* and *CD* can be found. Let *d* be the point in which the vertical through *D* cuts the horizontal plane containing the base line *AB*: then in the triangle *DAd* the length of *AD* has been found, the angle *AdD* is a right angle and the angle *dAD* can be measured, so that the lengths of the sides *Ad* and *Dd* can now be found. And since in the triangle *Acd* the lengths of the sides *Ac* and *Ad* have been found, then by measuring the angle *cAd*, that is, the angle between the vertical planes containing *AC* and *AD*, the length of the side *cd* can be determined.

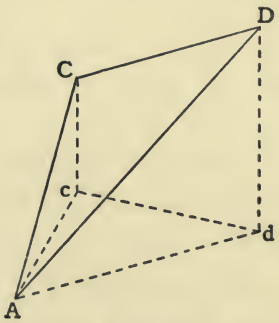


Fig. 6.

Proceeding in this way the lengths of all the lines connecting the points *A*, *B*, *c*, *d*, . . . in the horizontal plane through *AB* can be determined by calculation, provided the primary points *C*, *D*, . . . are visible from one or both of the points *A* and *B*.

To extend the survey two points *P* and *Q* must be selected from among those which have been

surveyed in relation to the base line AB. PQ is then taken as a new base line and a series of points are surveyed so as to fix their positions relatively to the horizontal plane through PQ.

Finally the survey is completed by referring all points not to the horizontal plane through AB but to sea-level. A spherical, or rather a spheroidal, surface is supposed to be drawn representing what

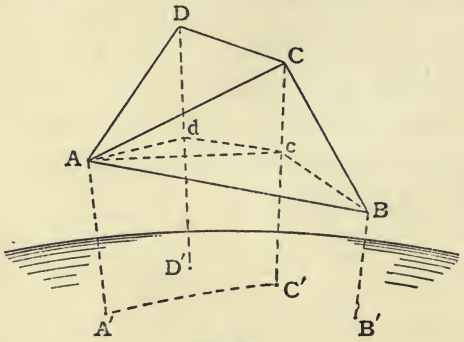


FIG. 6A.

the shape of the earth would be if it were in a fluid condition in a state of equilibrium; and every primary point of the survey is represented on this surface by the point in which the vertical through it cuts the surface. Suppose that A', B', C', ... are the points on this surface corresponding to the primary points A, B, C, ..., the relative positions of A', B', C', ... can be easily calculated; but when the base lines of the survey are selected at only moderate heights above the sea, the length of any line such as $A'C'$ does not differ appreciably from the length of the corresponding line Ac in the horizontal plane through AB. For instance, if the length of Ac

was 10 miles, and if the height of Ac above sea-level was 100 feet, then the length of $A'C'$ would be a quarter of a foot less than 10 miles.

In England the Ordnance* Survey was carried out by dividing the country into triangles in which the sides were between 30 and 40 miles long; and then these triangles were divided into a number of smaller triangles. A similar procedure has been followed in France and in other European countries. But in other parts of the world, such as Africa, where good maps have been required without much delay, the usual course has been to survey a chain of primary triangles across the country and then to complete the map by a series of secondary triangles.

In an official survey the whole series of primary triangles must be laid out with the utmost accuracy, which can only be ensured by applying checks at every stage of the work.

PLANIMETRY.

When the survey of a region has been completed the work involved in making a map consists of a double series of operations. The first series are concerned with the representation on the paper of all the points, lines, and curves which have to be shown; and the second series are concerned with the representation of the elevation of the different

* The word "Ordnance" is only applied to the Survey of England, because it was once under the *Ordnance Board*.

parts of the country. The former series are included under the term *planimetry*.

The chief difficulty in planimetry arises from the fact that a flat piece of paper cannot be wrapped smoothly on a spherical surface, as it can be wrapped on a cylinder or a cone. If a cylinder was made of thin metal it might be cut along a line parallel to its axis, and the metal might be pressed out flat without deformation. Similarly, a cone made of metal might be cut and the metal pressed out flat. But if a spherical ball made of thin metal was cut in any manner, no part of the metal shell could be pressed out flat without the use of considerable force, and then it would be found that the structure of the metal had been changed; and, if a picture had been drawn on the ball, after the metal had been pressed out the picture would be found to have suffered considerable distortion.

To construct on a plane or flat piece of paper a map which shall show all the details of the earth's surface with absolute accuracy is therefore impossible. But various methods have been suggested during the last five centuries which enable maps to be made that give fair representations of it, without much distortion. In all these methods the actual drawing of the map is reduced to a system of "plotting" the principal points, leaving the minor details to be filled in by freehand, much in the same way as graphs are plotted. For instance, if a line has to be inserted to show the course of a river, a few points only need be plotted, the rest of the line being drawn

by freehand. Just as in the case of other graphical work the labour involved in drawing a map is often simplified by the use of squared paper.

A very simple method of making a map is to plot the principal points by means of their geographical co-ordinates, latitude and longitude. On the paper on which the map is to be drawn take two straight lines OX and OY cutting at right angles. Let OX represent the equator of the earth, and let OY represent the meridian of Greenwich (or other fixed meridian from which longitude is measured). Then to determine on the map the position of a place P of which the latitude and longitude are known, measure off ON on OX to represent the longitude of P, and then draw NP perpendicular to ON to represent the latitude of P. The position of P is thus determined; and in the same way the position of any number of points may be determined.

In such a map the meridians and parallels of latitude would be straight lines parallel to OY and OX respectively. It would generally be advisable to draw a series of meridians and parallels first, and the work involved in plotting the principal points would be facilitated by using squared paper.

To insert on the map the course of a river it would be necessary to plot certain points on the river, such as the points where it is joined by its chief tributaries and the points where it changes its direction. These points could then be connected by freehand, and the course of the river clearly indicated.

In the same way roads, railways, canals, and political or other boundaries might be inserted.

It would be easy to draw a map by this method, but such a map would not be very accurate, except within 5° of the equator. Beyond this limit there would be considerable distortion, so that measurements made on the map in different directions would not be equally accurate. For instance, at a point whose latitude is 8°, while a length measured on the map along a meridian would give a correct distance on the earth, a length measured along a parallel of latitude would be too great by as much as one per cent. At a point whose latitude is 18° a length measured on the map along the parallel of latitude would be too great by as much as five per cent.

Although the method just explained is not of much practical utility, a slight modification of it furnishes a very useful method of filling up details on maps when the lines representing the meridians and parallels of latitude have been drawn. A simple example will make this clear. Say a map is required of that part of France which lies between latitude 50° and latitude 51° and between the meridians of which the longitudes are 2° and 3° east of Greenwich. Suppose that the meridians are drawn at intervals of 15′, and that the scale of the map is $\frac{1}{500,000}$, in which one centimetre represents five kilometres. It will be found that the meridians and parallels divide the map into rectangular spaces 5·6 cm. by 3·6 cm. Now suppose that it is required to insert on the map the railway station at Bapaume, of

which the latitude is 50° 5′ 52″ and the longitude is 2° 50′ 42″. This point lies within the rectangle bounded by the parallels 50° and 50° 15′ and by the meridians 2° 45′ and 3°. Let P denote the point

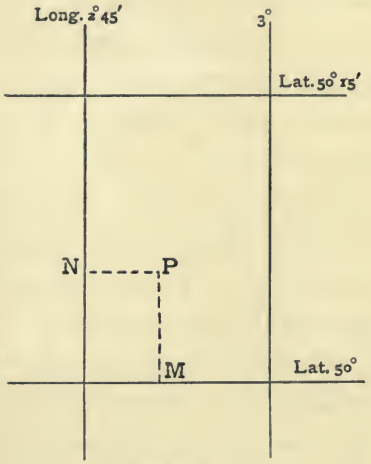


FIG. 7.

to be found on the map, and let PM, PN be drawn perpendicular to the nearest parallel and meridian. Then

since $2° 50′ 42″ - 2° 45′ = 5′ 42″ = 5{\cdot}7′$,

and $50° 5′ 52″ - 50° = 5′ 52″ = 5{\cdot}9′$;

$$PM = \frac{5{\cdot}9}{15} \times 5{\cdot}6 \text{ cm.} = 2{\cdot}2 \text{ cm.};$$

and $$PN = \frac{5{\cdot}7}{15} \times 3{\cdot}6 \text{ cm.} = 1{\cdot}4 \text{ cm.}$$

Thus the position of P is determined.

PRINCIPAL METHODS OF MAP-PLOTTING.

Some thirty different methods for drawing maps have been proposed which are of practical utility. Of these thirty methods there are six with which the soldier should be familiar. The first two are those generally used for topographical maps, that is, maps of relatively small areas, suitable for the use of an army operating in the field. The other four methods are those which have been employed in the construction of the military maps of countries like France, Germany, and Russia. In describing them "map" is used in a restricted sense to mean the preliminary process, and where the meridians and parallels are drawn the *reticulation* or network they form.

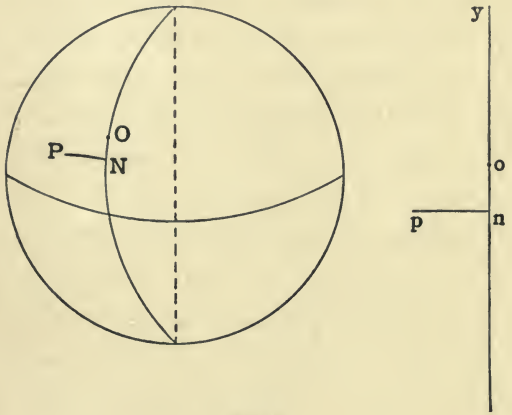
A simple and easily constructed system of map plotting is the *polyhedric*. The spheroid is here divided into a large number of adjacent quadrilaterals, each bounded by two parallels and two meridians. Each of these quadrilaterals makes one sheet of the map and is considered as coinciding with the plane that is at a tangent to its central point.* But it is impossible to place many of these sheets side by side to form a combined map, as they represent the faces of a polyhedron which is not developable as a whole on a plane, that is to say, it cannot be spread out flat.

1. *The Rectangular Co-ordinate Map.* A meridian is selected as the central or principal meridian

* The largest surface that can be treated as a horizontal plane without appreciable error is a square of 100 kilometres or 62½ miles; the actual error on the ground being 2·88 metres or about 3 yards.

of the country or region which is to be mapped, and a fixed point *O* is taken on it. From any point *P* a perpendicular *PN* is drawn to this central meridian, and the distances *PN* and *ON* are measured.

To draw the map, take *o* to represent the point *O*, draw the straight line *oy* to represent the meridian through *O*; measure off *on* along *oy* to represent



Fig' 8.

ON, draw *np* perpendicular to *on* and measure off *np* on the same scale as *on* to represent *NP*. In the same way any number of points *a*, *b*, *c*, ... can be plotted on the map to represent points *A*, *B*, *C*, ... on the earth.

This is the method used for constructing the British Ordnance maps.* It is a method which may be used for making a map of a region stretching to *any extent in latitude*, but it should not be used to include places

* It was originally introduced by Cassini for the construction of the general map of France, which bears his name and is dated 1740. (See opening of Chapter IV.)

which are more than 150 miles distant from the central meridian.

When this method is employed for mapping a topographical survey, it is not usual to insert the meridians or parallels of latitude, which would be curves and not straight lines or circular arcs.

2. *The Central Map.* A point *O* is selected somewhere about the centre of the region to be mapped. Then the distance of any point *P* from *O* is measured, and also the bearing of *P* from the north, that is, the angle which *OP* makes with the meridian of *O*.

To draw the map, take *o* to represent the point *O*, and draw the straight line *oy* to represent the meridian of *O*. Then draw the straight line *op*

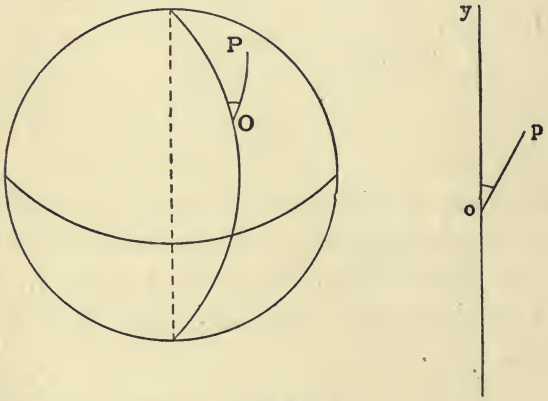


Fig. 9.

making with *oy* the angle *yop* equal to the angle which *OP* makes with the meridian of *O*, and measure off *op* to represent *OP*. In the same way any number of points can be plotted.

This method was originally proposed for a polar

map, for which purpose it is best. It is also known as the "equal-distance Zenithal" map.

For topographical purposes, it should not be used to include places which are more than 200 miles from the centre.

The Central Map is closely related to, and might be called a modification of, the Stereographic Map, which is one of a series of *perspective* maps that are derived from the mathematical theory of *projection*. These maps were more frequently used in the past than they are to-day, particularly as Atlas Maps of continents. Hence it has been the custom to speak of all maps as *map projections*; but, except in the case of the perspective class of maps and of one or two special maps, there is no connection between the methods employed for drawing maps and the mathematical theory of projection. The general use of the term "map-projection" has consequently been avoided here, in order not to cause a confusion of ideas later on.

The best of the perspective maps was produced by Colonel A. R. Clarke and is known as Clarke's *Minimum error perspective projection*. Its essential principle is that the necessary error at any point shall be as small as possible.

In the stereographic map every circle on the earth, great or small, is represented by a circle, and all angles are reproduced correctly. Except however for maps of the polar regions, this method has been little used, because in practice it is troublesome to draw the meridians and parallels.

The methods which have just been explained are not suitable for France or Russia. For those countries the best maps are constructed either on what is known as the *Conical* method, or on the *Polyconical* method. These methods were suggested by the fact that, if a cone is constructed to touch the earth along a parallel of latitude, the surface of the cone within a short distance on either side of this parallel is very nearly identical with the surface of the earth near the same parallel. Various modifications have been proposed by map-makers which make it possible to obtain fairly accurate maps for wider limits. The most useful of these methods are known as the Simple Conical Map, Bonne's modified Conical Map, and the Polyconical Map, with the modification of it known as the International Map. In plotting all these maps it is best to begin by drawing the meridians and parallels of latitude.

3. *The Simple Conical Map.* Let N be the north pole of the earth. Let O be some central point in the country which is to be mapped, then ON is taken as the central meridian and the parallel through O as the standard parallel. Let the tangent to the meridian NO at O cut the polar axis of the earth in T. Let NHF be any one of a series of meridians cutting the parallel through O in H; and let FG be one of a series of parallels of latitude cutting the meridians NO and NH in G and F respectively.

To plot the map on paper take the point o to represent O, and draw the straight line oy to represent the meridian of O. On oy measure off ot to represent

the length of *OT*. With centre *t* and radius *to* draw the arc *oh* to represent the parallel of latitude through *O*. On *to* measure off *og* to represent the length of the arc *OG* measured on the earth; with centre *t* and radius *tg* draw the arc *gf* to represent the parallel *GF*. Then take *h* in the arc *oh* so that the length of the arc *oh* represents the length of *OH* measured

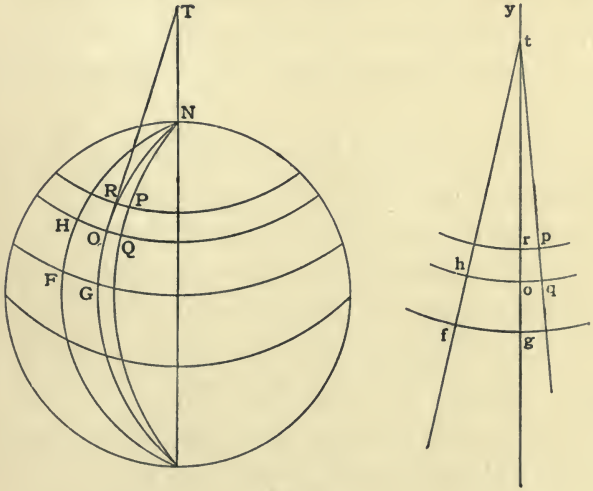


Fig. 10.

on the earth and draw the straight line *thf* cutting the arcs *oh* and *gf* in *h* and *f* respectively; *tf* will represent the meridian *NHF*. In the same way all the other parallels of latitude and meridians can be drawn. The parallels will be concentric arcs having *t* for centre, and the meridians will be straight lines radiating from *t*.

Now let *P* be any point on the earth; suppose the meridian of *P* cuts the parallel through *O* in *Q*, and suppose the parallel of latitude through *P* cuts the

meridian of O in R. To plot on the map the point p corresponding to P, measure off or along ot representing the length of OR, and take q on the arc oh so that the length of the arc oq represents the length of OQ measured on the earth. Then by drawing the straight line tpq and the arc rp with centre t the position of the point p is determined.

This map is not suitable for a country which extends between wide limits of latitude, but it is very good for one having wide limits of longitude, such as Canada.

4. *Bonne's modified Conical Map.* As in the last case, let some central point O be taken in the country which is to be mapped, and take the meridian ON of O as the central meridian. Let the tangent at O to the meridian ON cut the polar axis of the earth in T; let FG be any one of a series of parallels of latitude cutting the meridian ON in G; and let NHF be any one of a series of meridians cutting the parallels through O and G in H and F.

To draw the series of parallels of latitude on the map the method is exactly the same as in the last case. Take o to represent O, and draw the straight line oy to represent the meridian ON; on oy measure off ot to represent the length of OT, and measure off og to represent the length of OG measured on the earth. Then the arcs with centre t with radii to and tg will represent the parallels OH and GF. In the same way the lines on the map which represent other parallels of latitude can be drawn, and, as before, these lines will be concentric arcs with t for centre.

In this map the lines representing the meridians will not, however, be straight lines passing through *t*, with the exception of the central meridian through *o*. These lines are not simple curves which can be drawn, but have to be *plotted*. To plot the line which is to represent the meridian *NHF*, on the arc *oh* representing the parallel *OH* take *h* so that the

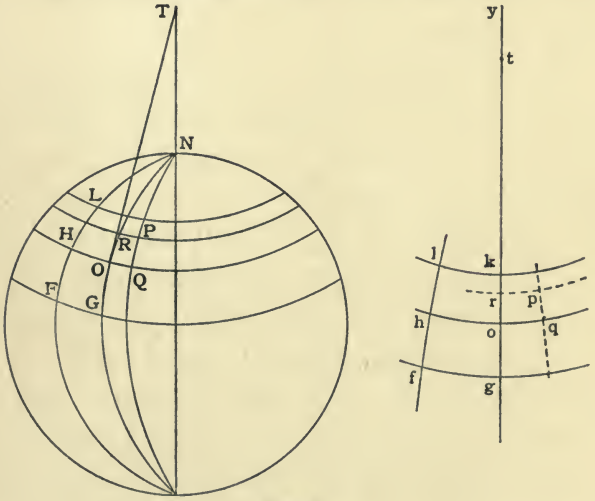


Fig. 11.

length of the arc *oh* represents the length of *OH*, and treat all the arcs representing parallels in the same way. Thus, on the arc *gf* take *f* so that the length of the arc *gf* represents the length of *GF*; and similarly *l* can be found on any other arc *kl*. When the points in which the line representing the meridian *NHF* cuts all the arcs *oh*, *gf*, *kl*, ... have been plotted, the line *lhf* representing the meridian can be drawn by freehand.

The position of the point on the map which corre-

sponds to any point *P* on the earth may be plotted by the same method as in the case of the conical map, that is, by drawing the lines which represent the meridian and parallel through *P*.

Bonne's map is the one which has been chiefly used in France, Russia, and other parts of Europe. It is more suitable for a country covering a wide range of latitude than for one that covers a wide range of longitude.

5. *The Polyconic Map.* As in the last two cases a central point *O* is selected and the meridian *ON* is taken as the central meridian. On the map *O* is represented by the point *o* and *ON* is represented by the straight line *oy*. The parallel of latitude through *O* is, as before, represented on the map by the arc drawn with its centre *t* on *oy* such that *ot* represents the length of the tangent *OT*.

The lines on the map which represent the other parallels are drawn in the same way. Thus to draw the line representing the parallel through a point *G* on the meridian *ON*, on the line *oy* take *g* so that *og* represents the length of *OG*, and then measure off *gs* to represent the length of *GS*, the tangent at *G* to the meridian *NOG*. Then with centre *s* and radius *sg* draw the arc *gf*, this will represent the parallel through *G*. These lines are, therefore, no longer concentric arcs.

The lines which represent the meridians on the map are drawn by the same method as that used in Bonne's map. That is, the line for the meridian *NLHF* is drawn so as to cut the lines *gf*, *oh*, *kl* ... in

points *f*, *h*, *l*, ... such that the lengths of the arcs *gf*, *oh*, *kl*, ... respectively represent the true lengths of *GF*, *OH*, *KL*, ...

This map is not very suitable for a country extending between wide limits of longitude. But it would be suitable for wider limits of latitude than Bonne's map.

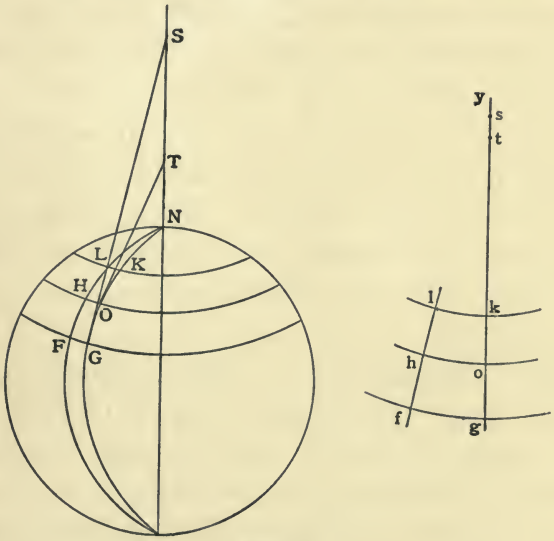


Fig. 12.

In constructing maps by the three methods which have just been described, it is usual to begin by drawing the lines which represent the meridians and parallels of latitude. This is an operation which is much facilitated by using what are known as *geodetic* tables, in which the length in miles or kilometres of 15′ of latitude and longitude is given at intervals of 15′ for all latitudes. The length of the tangent to the meridian is also tabulated for the same intervals of latitude.

The lines representing the meridians and parallels divide the map into a network of areas which are called *graticules*. If the map is on a large scale, such as the British one-inch maps where the meridians and parallels are given for intervals of 10′, the graticules are practically rectangles.

When the meridians and parallels have been drawn the principal points in each graticule are generally inserted by the method which has already been explained (p. 21) by means of the geographical co-ordinates of the points, as given by the survey on which the map is based. Other details, such as the rivers, roads, canals, and railways, are filled in by freehand from the sketches made in the course of the survey.

6. *The International Map.* This modification of the polyconic map was recommended by the International Map Committee, which met in London in 1909, as the most suitable for a series of sheets where each sheet is to fit the adjacent sheets. According to the directions of the committee the International Map on the scale of 1 to 1,000,000 is to consist of sheets bounded by parallels that differ by 4 degrees of latitude and by meridians that differ by either 6 or 12 degrees of longitude as the latitude is less or greater than 60°. Firstly the two extreme parallels for any sheet are drawn by the same rule as in the polyconic map, and then the points in which these parallels intersect the meridians are marked as before. The meridians, however, are now drawn as straight lines connecting these points. The object of making the

meridians straight lines instead of curves as in the polyconic map was that each sheet should fit the sheet adjacent to it on the East and West.

In a sheet drawn as described only the central meridian will be of correct length, while the other meridians are slightly too long, as in the case of all polyconic maps. To remedy this defect as much as possible the committee suggested that the distance between the extreme parallels should be so reduced that a correct length was given to the meridians 2° of longitude on either side of the central meridian. And tables were published in 1910, which make the work of inserting the meridians and parallels for any sheet quite simple.

The new coloured map of France on the scale of 1-50,000 which the *Service géographique de l'Armée* has brought out during the last few years is constructed on the same lines as the International Map. Each sheet covers 40' of longitude and 20' of latitude, while the design is made to overlap somewhat on the East and West. The ellipticity of the earth is taken into account, Colonel Clarke's latest calculations of this fraction, 1/293·465, having been adopted.

MAPS IN GENERAL.

It has already been stated that it is impossible to draw a map which is a perfectly accurate representation of any portion of the earth's surface. If a perfect map could be drawn, the scale would be the same for all parts of the map, that is, if a length

of one inch at any part of the map represented 10 miles on the earth a length of one inch would represent 10 miles at all parts of the map. There is, however, no map which is perfect in this respect. But it is possible to draw a map in which angles are accurately given; and it is possible to draw maps in which the shape or form of any small part of a curve on the map is exactly similar to the shape of the corresponding part of the curve on the earth's surface which it represents—such maps are said to be *orthomorphic*, that is, of the right form. There are also several maps in which areas are correctly represented, that is to say, the area of a lake or island is correctly given although the form of its boundary is not correct—such maps are called *equal-area* maps. A map cannot be orthomorphic as well as equal-area, for then it would be a perfect map, of the same scale throughout. Maps, indeed, exist in which the scale varies very slightly at different points; but it is impossible to draw one in which the scale is everywhere exactly the same.

There are some thirty different kinds of maps in use. All of them have certain advantages, but many of them are quite unsuitable for military purposes. For instance, Mercator's map, which is the best for nautical purposes, has many advantages. It is orthomorphic, the scale at every point being the same in all directions; the meridians and parallels are straight lines cutting at right angles; but its chief value is due to the fact that a curve on the earth which cuts all the meridians at a constant angle is represented

on the map by a straight line. As this so greatly simplifies "sailing by course," Mercator's map is universally used at sea. For statistical purposes it is advisable to use an equal-area map, and it is quite immaterial that the map used for such purposes should be orthomorphic, or that the scale should be everywhere the same.

For military purposes it is very desirable that any error caused by using the same scale should be as small as possible. In the map drawn by the rectangular co-ordinate method the error at a distance of 150 miles from the central meridian does not exceed 1/1000; and in the case of the central map the greatest error does not exceed this fraction at a distance of 200 miles from the centre. Either of these methods is well adapted for field work. The central map is probably the best to use for a single sheet map, but the rectangular co-ordinate method is most generally used, particularly in large cadastral surveys.

For a topographical series of maps, where the sheets are not intended to fit accurately together, the system generally used is the polyconical, each sheet being constructed with its own central meridian. For a series in which the maps are intended to fit together, the system generally used has hitherto been a modification of the conical map; but in future that of the International Map will doubtless be universally adopted.

I.

PLAN AND ELEVATION.

THE positions and outlines of the chief places, boundaries, and ways of communications in a district, as well as the meridians and parallels, are plotted from an ideal smooth surface, which is imagined to extend over the whole earth at mean sea-level, on to the map paper by one of the methods already described. The actual difference in altitude between any two points in this district, therefore, remains to be shown; and that can now be done by indicating on the map the height and shape of all hill features. Before describing the symmetrical method of representing form usually adopted in military maps, some explanation of the elementary geometrical considerations that are involved may be required.

GEOMETRICAL PROJECTION.

The foot of the perpendicular let fall from a point on to a horizontal plane is called its horizontal projection.

Thus, the horizontal projection of the point in space A is a, the foot of the perpendicular let fall

on the horizontal plane M, called the plane of projection.

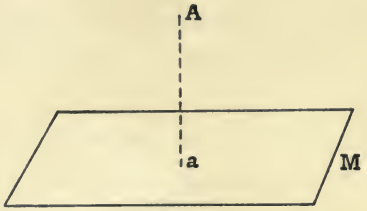


FIG. 13.

The projection of a straight line is obtained by joining the projections of its extremities. Thus, the horizontal projection of AB is ab; and, if C be in the same vertical plane as AB, the horizontal projection of AC is ac.

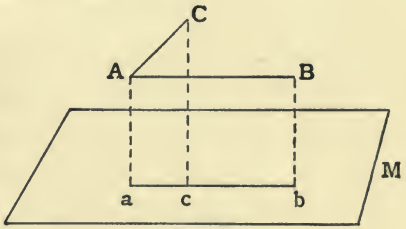


FIG. 14.

The projection of any figure is obtained by joining the projections of all the points in it.

Thus, the horizontal projection of the curve $ABCD$ is $abcd$. It will be noticed that figures are only

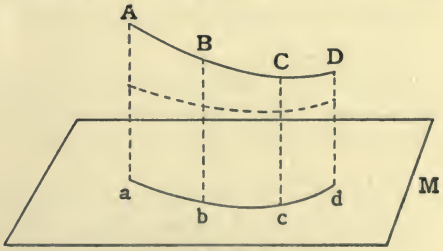


FIG. 15.

projected in their true size when they lie on a plane parallel to the horizontal plane, all others being more or less modified. An ellipse in a plane oblique to the horizontal plane may, for instance, be projected as a circle.*

In the topographical map the different points on the surface of the ground are represented, not by a reduced figure exactly similar to the country, which would be impossible on the flat, but by a figure similar to that given by the horizontal projection of these points. This figure is called the planimetry (*la planimétrie*), or ground plan. The representation of the form of the surface so as to show its relative elevation constitutes the *nivellement.*

Any horizontal plane might be chosen as the plane of projection for the planimetry of a given area. For whatever such plane be taken the projection of the same straight line will always be equal to that in any other horizontal plane, since all are parallel and contained between parallels, nor will its direction be altered.

But this is not so for the elevation (*nivellement*), in which case, in order that all points in the given area may be conveniently projected, it is necessary that the horizontal plane chosen should be situated

* The general definition of projection may be stated thus: "If from all the points of any figure perpendiculars be let fall on any plane their feet will trace out a figure which is called the *projection* of the given figure."

The area of any figure in a given plane bears a constant ratio to the area of its projection on another given plane. This ratio, in fact, depends solely on the angle between the planes.

beneath the lowest of them. For this reason it has been decided to adopt the mean sea-level as that of the plane of projection.

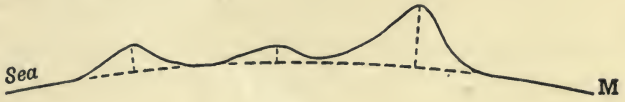


Fig. 16.

For great portions of the earth this imaginary surface is, of course, to be treated as being spherical, but it may be considered flat for regions of small extent.

THE METRICAL SYSTEM.

Always logical, the French have been at great pains to make the metre, which is equal to 39·37 inches, a natural rather than an arbitrary measure. It has been fixed, therefore, by observation as the ten millionth part of a quadrant of the meridian of Paris.

The quadrant (fourth part of a circle or angle of ninety degrees) is divided in France into 100 grades, each of which contains 100 minutes and each minute 100 seconds, so that while the minute is equal in length to a kilometre the metre is the tenth part of a geographical second. And these decimal measurements of latitude and longitude are used in the French military map. Now a kilometre is equal to 1093·633 yards, or nearly 1100 yards, which is five furlongs, so that a kilometre is $\frac{5}{8}$th of a mile, less six yards one foot.

Also 10 metres = about 10·94 yards,

or 1000 centimetres = 10·94 × 36 inches.

Therefore 5 cm. = $\frac{10 \cdot 94 \times 18}{100}$ inches

= 1·96 in., or nearly 2 in.

So that a centimetre = about $\frac{2}{5}$th of an inch.

Roughly, then:

$\frac{1}{4}$ of a millimetre = $\frac{1}{100}$ of an inch,

a millimetre = ·04 or $\frac{1}{25}$ of an inch,

a centimetre = ·4 or $\frac{2}{5}$ of an inch,

a metre = 3′3$\frac{1}{3}$″, or over 3$\frac{1}{4}$ feet.

30 centimetres nearly = 1 foot.

10 metres = 11 yards.

a kilometre = $\frac{5}{8}$ of a mile.

When these approximations have been borne in mind for a little while, they are remembered unconsciously.

And, although Mr. F.'s Aunt insisted that there were milestones on the Dover road, it is better to leave them behind when bound for France, where one soon learns to think in kilometres.

SCALES.

Only when the ground is absolutely flat will the map be an exact miniature reproduction of the country. For the rest, it will not be like the surface one sees, but similar to the figure which would be obtained if all the lines in it were projected on a horizontal plane.

If l, l', l'', etc., are the lines of the map that corre-

spond with the projected lines of the ground L, L', L'', etc., then :

$$\frac{l}{L}=\frac{l'}{L'}=\frac{l''}{L''}= \text{ etc.}$$

will always be a constant ratio, or equal to the same fraction, which is consequently termed the *representative* fraction. This invariable relation of all the lines of the same map to those of the ground when projected is called the *scale* of the map in question.

The scale of a map is therefore the constant ratio which any line of the map bears to the horizontal projection of the corresponding line in the country, or, what comes to the same thing, the scale of the map is the constant proportion to which all the elements of the planimetry of that map are reduced.

The scale is expressed numerically in the form of a fraction, of which the numerator is 1 and the denominator in the metrical system a multiple of a thousand.

Thus, if M represents the required multiple of 1000 :

$$\frac{l}{L}=\frac{l'}{L'}=\frac{l''}{L''}=\frac{1}{M}.$$

Thence are derived two useful formulas

$$(1) \quad l=\frac{L}{M}.$$

$$(2) \quad L=l\times M.$$

By their help one can compare lines on the ground with those in the map, and *vice versâ*.

The elementary problems involved are :

1. The length of a line on the ground being known, to find the distance that corresponds with it in the map.

Take a line of 500 metres on the ground. If the scale of the map be, for instance, 1 to 10,000, formula (1) becomes :

$$l = \frac{500 \text{ m.}}{10{,}000}$$
$$= 0{\cdot}05 \text{ m. or } 5 \text{ centimetres.}$$

2. The length of a line in the map being known, to find the distance that corresponds with it on the ground.

If a length of, say, 0·035 m. (or 35 millimetres) be measured on a map of which the scale is 1 in 50,000, formula (2) gives :

$$L = 0{\cdot}035 \text{ m.} \times 50{,}000$$
$$= 1750 \text{ m.}$$

These little conversions, simple as they are, demand a certain amount of time and may lead to some error. To obviate the necessity of such calculations a graphic method has therefore been devised.

The principle on which this method is based may be illustrated thus :

If it is agreed to represent AB, a line on the ground, by the line ab in the map, any line on the ground, $A'B'$, that is double the length of AB, will be properly represented in the map by $a'b'$, which is twice as long as ab.

Lines three, four, five, etc., times as long as AB

will be represented by the same multiples of *ab*. So that if *ab* is the 10,000th part of *AB* any line divided into parts that are each equal to *ab* becomes a graphic representation of the scale of 1 in 10,000.

That is to say, if $AB = 10{,}000$ cm. (or 100 metres) and $ab = 1$ cm., the graphic scale of 1/10,000 is obtained by measuring off on a line *MN* the lengths *ab*, *bb'*, *b'b''*, etc., all equal to 1 centimetre. Ten divisions, for instance, would represent a length

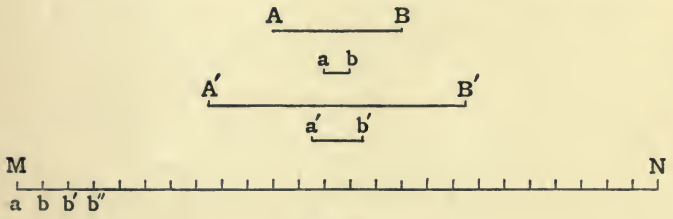


Fig. 17.

ten times as great as *AB*, or a kilometre. And this proves correct when tested by formula (1).

$$l = \frac{1000 \text{ m.}}{10{,}000} = 10 \text{ cm.}$$

Suppose one has to make a graphic scale of 1/80,000, when 1 metre in the map will correspond with a length of 80,000 metres on the ground, that is to say, when 1000 metres are represented by the eightieth part of a metre, or 0·0125 m. ($1\frac{1}{4}$ cm.).

It is usual to make the length of the scale about two-thirds of the greatest distance one will need to measure. If this is 8 cm. the length of the scale will be between 5 and 6 cm.

On a straight line of indefinite length take *AB*

equal to 0·05 m. which represents 4000 m. Above or below the point *B* write this number and the cypher 0 above or below the point *A*. Then draw any straight line *AC* that makes an acute angle with *AB* and measure off on it four sections of the same length, *Aa*, *ab*, *bc*, and *cd*.

Join the point *d* to *B*, and through the intermediate points *a*, *b*, *c*, draw parallels to *dB*. By the similarity of triangles the sides of which are parallel, the straight

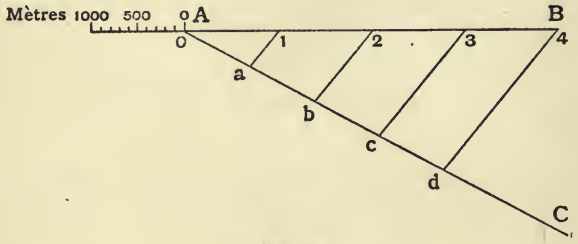


Fig. 18.

line *AB* will be divided in this manner, known as *graduation*, into four equal parts, each representing a length of 1000 m. One, therefore, writes the numbers 1000, 2000, and 3000, or simply the figures 1, 2, and 3, to denote kilometres above or below each division between *A* and *B*.

In order to measure lengths of less than 1000 metres, prolong *BA* by the length of one division. This extension to the left, consequently, represents 1000 m. Now divide it into ten equal parts, each of which will represent 100 m. Then above each division, starting from the point *A* as zero, one writes the numbers 100, 200, 300, etc., to 1000, or, as in the figure above, only 500 and 1000.

The part AB is the scale proper, the other part is called the *talon* or heel.

It goes without saying that if AB had been given a value different from 4000 m. the values of all the divisions would have been changed in the same ratio. Had AB represented 40 m., for instance, each division of the scale would have only corresponded to 10 m. and each division of the *talon* to 1 m., while the scale itself would then have been 1/800.

The relation

$$\frac{l}{L}=\frac{1}{M}$$

gives as a third formula (3):

$$M=\frac{L}{l} \text{ or } \frac{\text{length on ground}}{\text{length on map}},$$

and applied to the case above, where a division of the graphic scale measuring $1\frac{1}{4}$ cm. represents 10 m. or 1000 cm.

$$M=\frac{1000}{\frac{5}{4}}=1000\times\frac{4}{5}=800,$$

which makes the numerical scale 1/800, as stated.

The use of the graphic scale considerably shortens the two elementary operations of topography, as well as reducing the chance of error.

To find what will be the length on the map of a measured line on the ground one has only to open a pair of compasses until the points include the corresponding divisions on the scale and then to bring them into the indicated position on the map. Suppose that one has to ascertain a length which on a map of 1/10,000 corresponds with a length of

430 metres measured on the ground. Place one point of the compasses on the division of the scale marked 400 and the other point on that marked 0, which gives them an extension of 4 cm. Then advance the left-hand point to the division of the *talon* marked 30, which further extends the points by 3 mm. The required length is therefore equal to 4 cm. and 3 mm., that is, to 43 mm. or 0·043 m., which may now be correctly measured on the map.

The inverse operation is as easy, and will be more frequently performed.

In general terms it may be described thus: To find the length L that corresponds with a given length l measured on the map, extend the compasses until the opening is equal to l. Then place them on the scale so that one point rests on a division of the scale proper and the other overlaps a portion of the *talon*. If the left-hand point of the compasses falls exactly on a division of the *talon*, the sum of the two readings will give the length required. Should this not be the case one must guess the length included between that point of the compasses and the nearest division of the *talon*.

As, however, neither eyesight nor instrument is perfect, the smallest appreciable length has been found to be a quarter of a millimetre (the hundredth part of an inch). When one measures off a length on the scale, therefore, one commits a *graphic error* the actual equivalent of which is given by making $l = 0{\cdot}00025$ m. or $\frac{1}{4000}$ of a metre in formula (2).

Thus: $L = 0{\cdot}00025 \text{ m.} \times M$.

Hence, for every measurement made on a map of 1/10,000

$$L = 0{\cdot}00025 \text{ m.} \times 10{,}000$$
$$= 2{\cdot}50 \text{ m.}$$

In the scale of 1/80,000, which is used the most, as it has been adopted for the construction of the French General Staff map, the probable error would be given thus:

$$L = 0{\cdot}00025 \text{ m.} \times 80{,}000$$
$$= 20 \text{ m.}$$

One has, indeed, only to divide the scale denominator by 4000 to obtain it in metres.

It is scarcely necessary to point out that the approximate difference from a true reading of the scale is not affected by the length measured.

But as each measurement requires two operations, one on the map and one on the scale, or *vice versâ*, the error occurs twice. These deviations will augment each other if both are beyond or both below the true mark, and will tend to cancel each other where one measurement is too great and the other too small; so that the total error of observation will vary between half a millimetre too much and half a millimetre too little, on either side of the point when it is nil.

The approximations in the two examples given above should not, therefore, be 2·50 m. and 20 m., but 5 m. and 40 m.

Since, then, it is impossible to appreciate with accuracy lengths of less than a quarter of a millimetre,

no feature of the country should appear unless its representation in the map demands a greater length than this limit of measurement. But it will be seen later that this is not in fact always the case.

A topographical map to be serviceable must not be unwieldy. It should include a sufficiently large extent of country and show all the more important details to be met with there.

In the scale of 1/5000 a square the side of which is 5 kilometres will be represented by a sketch of 1 m. square, which is somewhat large for so small an area. One can scarcely, therefore, employ a bigger scale than this in topography. And, if details of 25 metres are not to be sacrificed one cannot employ a smaller scale than 1/100,000, in which the graphic error is approximately 25 metres.

The principal scales in use are :

$\frac{1}{5{,}000}$	Surveys of small areas, sketches of positions and plans of attack on fortresses.
$\frac{1}{10{,}000}$	Surroundings of a fortified town, special reconnaissances and short itineraries.
$\frac{1}{20{,}000}$	Longer itineraries, reconnaissances and certain original portions of the survey for the French General Staff map.
$\frac{1}{40{,}000}$	Original survey for the French General Staff map.
$\frac{1}{80{,}000}$	French General Staff map (*Carte de l'état-major français*).
$\frac{1}{100{,}000}$	Map prepared by Ministry of the Interior.

Scales above 1/5000, about 12·6″ to the mile, are called cadastral, and scales below 1/100,000, about 0·6″ to the mile, are called chorographic, while scales of less than 1/1,000,000, about two-thirds of an inch to ten miles, are called geographical.

The scale is usually placed on the lower margin of the design, the upper margin being reserved for the title or name of chief place in map.

PLANIMETRICAL ELEMENTS.

Generally speaking, all the important details on the surface of the ground, independent of their *relief* or height above the horizontal plane—railways, roads, streams, dwelling-places, woods, fields, etc.—are the planimetrical elements, and taken together constitute *la planimétrie.*

This is the view as seen from the car of a balloon, so high up that objects on the earth's surface appear to be all in the same plane ; and it is naturally more subject to change than is the conformation of the ground in these days.

Now, in the map of 1/80,000 no object under 20 metres in length and breadth can properly be shown, as its reduced dimensions would be less than $\frac{1}{4}$ of a millimetre. But if this principle were rigidly adhered to the map would be of little practical use, as railways, roads, streams, and buildings would not then be for the most part indicated. It has been found necessary, therefore, to adopt certain minute signs to represent important details which cannot be shown in their proper size.

These *conventional signs*, the size of which varies with the scale, though their primitive shape is preserved, have been chosen so as to represent in a general way the horizontal projection of the planimetrical elements to which they correspond. They are made large enough to be seen, but small enough not to obscure other details of the map.

The particular figures that always indicate the nature of certain planimetrical elements, even when their outlines are sufficiently extensive to be traced according to the scale of the map, are also called conventional signs.

Topographical maps being primarily constructed for the purposes of manœuvre and other operations of war, it is essential that the attention arrested by a conventional sign should be proportionate to the military importance of the detail that it represents.

The elements of planimetry are grouped in five divisions, thus :

1. Streams and everything connected with them. (Bridges, locks, etc.)
2. Ways of communication. (Railways, roads, etc.)
3. Beds of water-courses.
4. Towns, villages, isolated buildings, and various landmarks.
5. Indications of the quality of the soil, its mineral and vegetable products, with the boundaries that contain them.

The topographical map is supposed to be lighted by an oblique ray falling from the north-west at

an angle of 45°, and in the larger scales the shadow thrown is represented thus :

A mound An excavation

Fig. 19.

Tables of the conventional signs used for the different scales in France are issued, and should be studied in conjunction with the maps to which they belong.

FIXING THE ELEVATION OR NIVELLEMENT.

The object of *le nivellement* is to represent the form of the ground so as to exhibit all its undulations. To attain this the following conditions have to be satisfied :

1. To combine on the same sheet of paper the indications of the form of the ground with those of the planimetrical elements.

2. To give the representation of form an exact value comparable with that of the planimetry.

3. To appeal to the eye by the modelling of the ground.

Now any two points *A* and *B* on the surface of the earth, having as their horizontal projections or *planimetry* the two points *a* and *b*, will not generally be at the same height; that is to say, the lines *Aa* and *Bb* will not be equal in length.

The respective position or altitude of each of these points above the horizontal plane of projection

formed by a prolongation of the surface of the sea, which may be expressed numerically by the distance in metres that separates each of them from this imaginary "plane" is called its *cote* or altitude.

The *relief* of one point over another is the name given to the difference of *cote* of these two points. Thus, if the *cote* of *A* is 548 metres and *C* is another

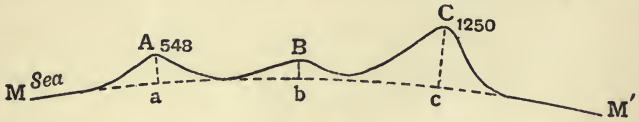


FIG. 20.

point the *cote* of which is 1250 metres, the relief of *C* over *A* is 702 metres. When the two points are visible from one another the relief takes the name of *commandement.*

FORMS OF THE GROUND.

Before studying the different means adopted to express the configuration of the ground it is necessary to examine the terms given to the various accidents and undulations met with on the surface of the soil.

The earth is supposed to have been originally an incandescent mass. In consequence of its having gradually cooled, the surface has solidified and a thin crust has been formed.* Owing to continued

* General Berthaut in his great work on the physical causes that have determined the present configuration of the earth, entitled *Topologie,* and published in 1913, sums up recent opinion on the condition of the interior of the spheroid thus: "The heavy substances of

loss of heat, the earth's crust has been subjected to contractions that have more or less violently changed its shape. Certain portions of the soil have sunk while others have been lifted, resulting in folds that have produced the various forms which constitute the relief of the ground.

Different parts of the earth's surface are either salient, that is, raised above the surrounding ground in mountains and hills, or re-entrant, that is, depressed into valleys. Between the two lie the flat lands which form the planes and plateaux.

Mountains are considerable upheavals of the ground, characterised by the abruptness of their sides and the absence of vegetation on their summits. Their height is never less than 300 metres.

Hills are not only lower but their slopes are gentler. They are cultivated on the top, and seldom exceed 300 metres in height.

Mounts are heights which vary greatly in form and are not usually more than 100 metres high.

Mamelons or knolls are isolated hills or mounts the form of which closely resembles the half of a sphere.

Buttes, *tertres*, or *mottes* are the names given to little hills that rise in a plane or plateau.

A *pli de terrain*, or fold in the ground, is a hill of an elongated form, the relief of which is scarcely

---

which the internal masses are composed are *probably* under the double influence of a very great heat and an excessive pressure, in a mixed state practically solid, that differs appreciably from the solid, liquid or gaseous state assumed by the various substances on the periphery."

noticeable. If the relief becomes accentuated it is called a *rideau* (curtain).

Everything that impedes the march of troops constitutes an accident, and every transformation that has moved the soil out of the horizontal is called a movement of the ground.

The ground is said to be *couvert* (covered) when trees, houses, or even crops prevent one from seeing any great distance ahead; and *coupé* (cut up) if ditches, hedges, walls, or brooks obstruct one's path. It is called *découvert* (open) if nothing interferes with either the line of march or sight.

A mountain has a base or foot, *flancs* (sides) or *versants* (water-sheds), and a summit or *faîte* (ridge).

The foot of a mountain is the point from which it begins to rise, and the highest part of the elevation is called the summit, which may be of some considerable extent.

The sides are the portions included between the foot and the summit. They are said to be in steps or terraces when they rise in a series of cliffs or escarpments, called *ressauts*, that are separated by almost horizontal surfaces.

The "reverse" of a height is the name given to the side opposite that under observation.

Isolated mountains rarely occur; they are either ranged together in a chain or form some irregular combination, so as to constitute a group (*massif*) about the central summit.

A chain throws out branches which when running

nearly at a right angle to its general direction are given the name of *contreforts* or buttresses.

The points at which these secondary chains meet the main chain are called knots.

A *croupe*, often formed like a donkey's back or crupper, is a little buttress that descends from such a knot into the plane between two more or less marked depressions.

A spur is a croupe with steep sides that springs out clear from a chain of heights.

The line that joins the summits of a chain is called *la ligne dè faîte* (ridge line), crest, *arête* (back-bone), or "parting of the waters."

The summits take different forms, called respectively *plateau*, *pic* (peak), *aiguille* (needle), *ballon*, *puy*, *dent*, etc.

The military crest, which must not be confounded with the ridge-line, is the line formed by joining all those points on a water-shed (*versant*) from which one can see the whole of the ground on that side, including the foot of the hill itself. This line is all-important for the defence of a position.

A valley may be defined as that part of the ground contained between two heights.

Cirques are vast and deep circular depressions. Their sides are generally steep, and only a rather narrow opening admits the water from the hills.

The flanks or water-sheds of a valley are the gentle slopes that continue the sides of the hills.

These slopes unite along a winding line that is called the *thalweg* or path of the valley, which is

usually the course of a brook or torrent. The term "water-course" may, therefore, be substituted with advantage for the German word "Thalweg," remembering always that portions of its bed are often dry.

Primitive valleys are those that have resulted from upheavals of the soil. They are called transverse or longitudinal as their general direction crosses or follows that of the chain of heights from which the water-course springs.

In order to connect the summits of a chain of heights the ridge-line must pass through a series of more or less marked depressions, called *cols*.

These cols, from their comparatively low level, form the best passages through the chain of mountains, just as the water-courses indicate the natural highways through the valleys.

Take, for instance, two summits *S* and *S'* divided by a col *C*. It is evident that the easiest and most

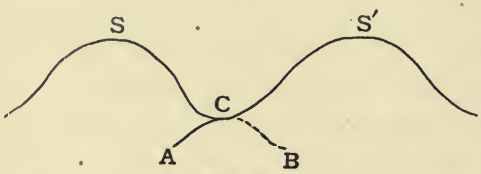


Fig. 21.

direct route from a point *A* situated on one watershed to a point *B* situated on the reverse passes over the col.

A defile (*défilé*) is, of course, any narrow passage that obliges troops on the march to contract their front. Thus bridges, streets, and footpaths are

called *défilés*; but the word is also used to denote a narrow col of any considerable extent.

A sudden narrowing in a valley is called *un étranglement*, and *gorge* is the name given to a very narrow and very deep valley the sides of which rise abruptly.

Planes, which only present a slight undulation and are generally near the sea, occupy the lowest parts of the valleys. Flanders is an example of a fairly large plane.

Plateaux, though also flat, being raised considerably above the sea-level, dominate the surrounding country.

Whether the ground be horizontal or not, however, it should be regarded as consisting of, firstly, forms and accidents due to natural causes, and, secondly, forms and accidents that are the work of man.

INCLINES.

The slopes that connect the mountains with the valleys are rarely inclined planes, but take rather the form of curved surfaces. Nevertheless, whatsoever be the form of any particular feature of the ground, one may consider it as being a polyhedron with an infinite number of faces that are perfectly flat and inclined at different angles to the horizontal plane. The angle so made by each of these faces is called its inclination.

The inclination of a straight line is measured by its gradient, that is, the fraction of which the numerator

is the difference of level of any two points on the line, and the denominator is the horizontal projection of the distance between them.

Thus the gradient of the straight line AB will be expressed by the ratio $\frac{BC}{AC}$

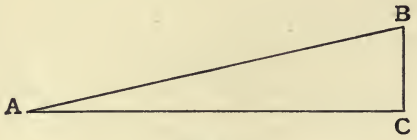


Fig. 22.

In France, AC is given the length of 1 metre, so that the gradient may be stated as so many centimetres in the metre, as well as fractionally.

If, therefore, BC is found to measure 4 centimetres $\frac{BC}{AC}=\frac{4}{100}=\frac{1}{25}$, and the gradient, sometimes expressed by the decimal ·04, is said to be 4 centimetres in the metre; or, as we should say, one foot in twenty-five.

The greater the angle a surface makes with the horizontal plane the greater will be its gradient.

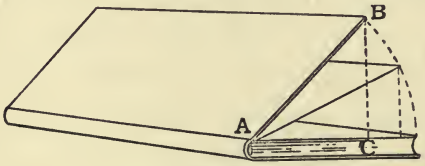


Fig. 23.

This will at once be seen by placing a book down on one cover and gradually opening the other. For BC constantly increases and AC constantly diminishes.

When a line is traced on a plane, whose position with respect to the horizontal plane does not change, which is the case with the ground, the greater the angle it makes with its projection the greater is its gradient.

It may be demonstrated geometrically that if a straight line *BA*, lying in the plane *MM*, is perpendicular to the straight line *MN* its projection *CA* is also perpendicular to *MN*.

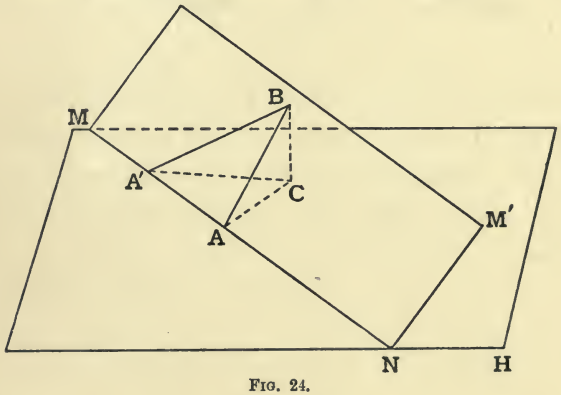


Fig. 24.

Consequently, if *BA'* is another line in the plane *MM'* its projection *CA'* will be oblique to *MN* and greater in length than *CA*. And since the gradient of the line *BA* is expressed by the ratio $\frac{BC}{CA}$ and that of the line *BA'* by $\frac{BC}{CA'}$, the numerators being the same, the second fraction is less than the first as its denominator is greater.

The line *BA* is then the steepest of all lines drawn in the plane *MM'* from the point *B* to the horizontal plane, and is therefore called a *line of greatest slope.*

It will be noted that one line of greatest slope is sufficient to determine the position of a plane, as all the others which can be drawn will be parallel and will make the same angle with the horizontal plane.

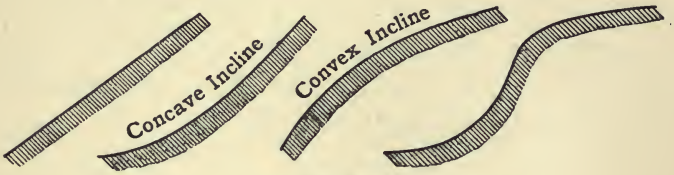


Fig. 25.

Inclines are termed straight, concave, or convex according to the appearance they present when seen in profile. But the sides of a hill are mostly convex in their upper part and become concave towards their base.

II.

CONCERNING CONTOURS.

A PLASTIC reproduction of the ground to scale would be the ideal map, but it would also be the least portable, and therefore of small utility in the field. Still, whether the relief of the ground is represented by a solid model or by a topographical plan, it is upon the same geometrical principles.

REPRESENTATION OF FORM.

It will be remembered that:

1. The projection of a point on a plane is the foot of the perpendicular let fall from that point on the plane.

The horizontal projection of the point *A* is *a*.

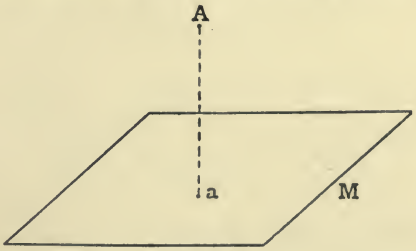


FIG. 26.

2. The projection of any line, straight, curved, or bent is formed by joining the projections of all the points in it.

One obtains this result approximately by joining the projections of the principal points in the line in question; two points being sufficient to determine the projection of a straight line.

3. The more a finite line is inclined to the plane of projection the shorter will be its projection.

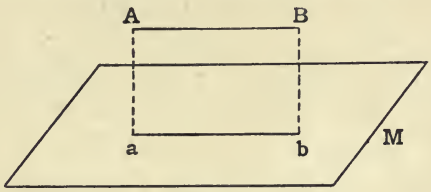


Fig. 27.

Let *AB* be a straight line parallel to the plane of projection and *ab* be its projection.

AB and *ab* will be equal and parallel, being opposite sides of a rectangle. That is, *AB* will be projected in its true size.

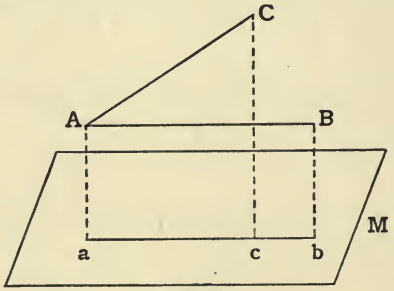


Fig. 28.

If *AB* turns round the point *A*, without leaving the vertical plane in which it was before, until it occupies the position *AC*, the figure shows that *ac* will be less than *ab*. So, if *AB* turned on in the same plane till it was perpendicular to the plane of

projection, its projection would be reduced to the point *a*.

Any point *a*, taken in the horizontal plane, may then be considered as the projection of some corresponding point *A* in space. But it does not indicate the position of *A* above that plane, since every point in the vertical line *Aa* has the same projection *a*. In order to fix the position of *A* we must, therefore, know the height of this point above the horizontal plane, or the length of the line *Aa*. It is this distance expressed in metres that constitutes the *cote* of the point *A*. The number representing this *cote* is written by the side of the point *a*. Thus, if the *cote* of the point *A* is 15 metres, the projection of *A* is denoted by a_{15}.*

*The position of any point *A* in space may also be fixed by rectangular co-ordinates.

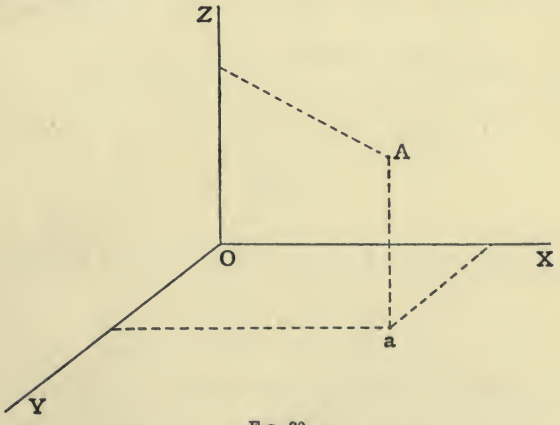


Fig. 29.

Thus reference to the axes *OX* and *OY* will give the position of *a* in the horizontal plane and a third axis *OZ* at right angles to the plane in which the other two lie will serve to determine the height of *A* above it, *Aa* being represented by *z*.

The projection of a straight line is represented by the projections of its two extremities together with their *cotes* ($a_m b_n$). If the straight line is horizontal, these two projections will have the same *cote* ($a_4 b_4$), for instance. The projection of a bent or zig-zag line is represented by the projections of the extremities of all the elements that compose it, say a_4, b_8, $c_{6\cdot25}$, $d_{9\cdot50}$.

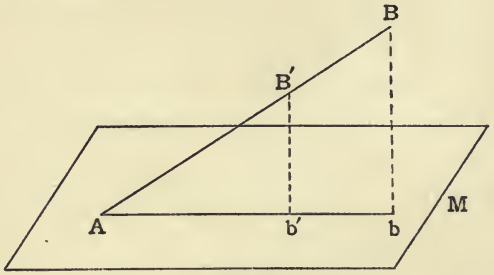


Fig. 30.

It has already been stated that the gradient of a straight line such as AB is expressed by the ratio $\frac{Bb}{Ab}$.

The gradient of AB' will then be expressed by

$$\frac{B'b'}{Ab'}.$$

Therefore, since by the similarity of the triangles

$$\frac{Bb}{Ab} = \frac{B'b'}{Ab'},$$

the gradient of AB is constant. Also the heights of points taken on an incline are proportional to the projections of the distances by which they are removed from its base.

Put in general terms, if h, h', etc., represent the

heights of Bb, $B'b'$, etc., and p, p', etc., represent the projections Ab, Ab', etc., then

$$\frac{h}{p} = \frac{h'}{p'} = g,$$

where g represents the gradient of the straight line AB.

HOW A VERTICAL FIGURE MAY BE MEASURED ON THE HORIZONTAL PLANE.*

Suppose that a plane P be turned on its axis of intersection XY with the horizontal plane H until it lies flat on that plane, in the same way that a book is closed.

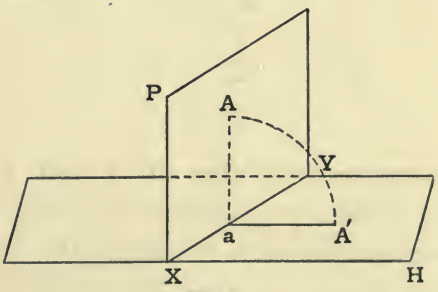


Fig. 31.

Any figure traced on the plane P would then be impressed in its true size upon the plane H, and the points of which it is composed would remain at the same distance from the hinge line XY.

Take a point A, for instance. If the plane P is vertical, a, the horizontal projection of this point,

* A general explanation of this method, called *rabattement*, will be found in the article on Geometry in the *Encyclopædia Britannica*, under the sub-heading of "Descriptive Geometry."

will be situated on XY, and the line Aa will be perpendicular to XY. When the plane P is turned through a right angle the lines XY and Aa, being both in this plane, will not change their relative positions or their lengths, and the point A will finally rest at A' on the plane H. $A'a$, which is only the last position of Aa, will, therefore, be equal to Aa and like it perpendicular to XY.

This furnishes a solution of various problems:

1. The projection of the straight line AB being known, to find its length and the angle it makes with the plane of projection.

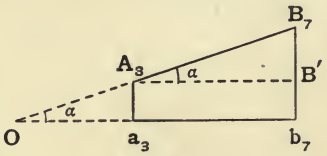


Fig. 32.

Take the respective *cotes* of A and B as being 3 and 7; and their projections as a_3 and b_7.

Draw perpendiculars to scale on the plane of the paper which represents that of projection and join A_3B_7. If the figure were to turn about the line a_3b_7 as a hinge until it became vertical, the line A_3B_7 would occupy the exact position of the given line in space. That is to say, it is traced on the paper in its true size, and its length may be measured there with the compasses.

Also if B_7A_3 and b_7a_3 be produced until they meet in O, the required angle has been found.

But if A_3B' be drawn parallel to a_3b_7 the angle

B_7A_3B' equals the angle B_7Ob_7. Consequently, in practice, it is unnecessary to produce the incline which would take up too much space when the *cotes* of A and B were either very large or nearly equal.

2. The projection of a straight line AB being known, to find the projection of the point on it which will be a given number of metres above the plane of reference.

Suppose the number to be 8, and the projection of the line $a_6b_{11\cdot7}$.

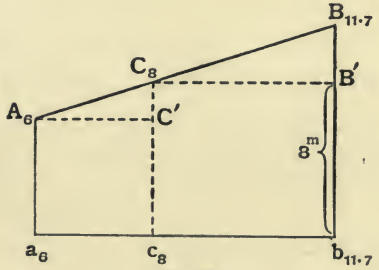


FIG. 33.

If a scale length of 8 m. is measured off from $b_{11\cdot7}$ to B' on the perpendicular $B_{11\cdot7}b_{11\cdot7}$ and a straight line be drawn from B' parallel to $a_6b_{11\cdot7}$, it will cut AB in a point C that is 8 m. above the plane of reference.

This point may, however, be determined in another manner.

Since
$$\frac{C_8C'}{A_6C'} = \frac{B_{11\cdot7}B'}{C_8B'},$$

or
$$\frac{8-6}{a_6c_8} = \frac{11\cdot7-8}{c_8b_{11\cdot7}},$$

That is,
$$\frac{2}{3\cdot7} = \frac{a_6c_8}{c_8b_{11\cdot7}}.$$

Therefore $a_6b_{11\cdot7}$ must be divided into two parts proportional to the numbers 2 and 3·7, obtained by subtracting the lower *cote* (6) from that of the given point C whose projection is required and by subtracting the *cote* C from the higher *cote* (11·7).*

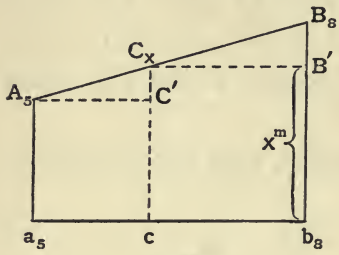


Fig. 34.

3. The projection of a straight line AB being known, to find the *cote* of the point in space that corresponds to a point c on it.

If the projection be, for instance, a_5b_8, a perpendicular cC_x having been drawn,

* Any straight line M may be divided into two parts that are respectively proportional to two given lengths or numbers N and P by the familiar method of graduation.

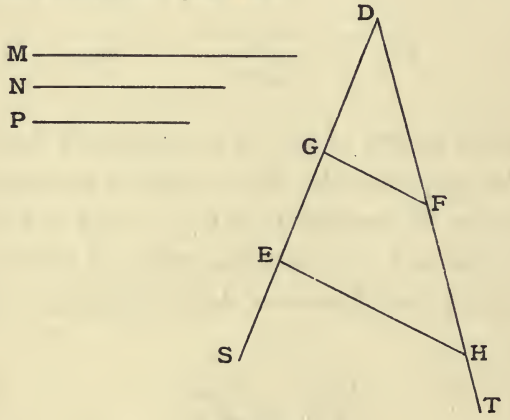


Fig. 35.

Take any line DS of indefinite length and DT any other line making a convenient angle with it.

On DS measure off DE equal to given straight line M, and on DT measure DF equal to N and FH equal to P. Join HE and draw FG parallel to it. DE will then be divided at G in the required proportion.

and C_xB' parallel to a_5b_8, the length of b_8B' will be the required *cote*.*

METHOD OF CONTOURING.

As the position of any point in space is fixed by its projection on the horizontal plane and its *cote*, the form of the ground may be represented by a sufficiently large number of these combined indications or co-ordinations.

To prevent confusion and to secure a picture that is easily understood as well as complete, the ground is supposed to be cut by a series of equidistant planes parallel to the horizontal. The intersections these planes make with the surface of the soil are projected on the horizontal plane.

In each of these intersections, called "contours" (*courbes de niveau*), every point being at the same height above the horizontal plane, the whole line need only be marked with one *cote*.

* It may be found numerically thus: Draw A_5C' also parallel to a_5b_8.

Then $$\frac{B_8B'}{C_xB'}=\frac{C_xC'}{A_5C'},$$

or $$\frac{8-x}{cb_8}=\frac{x-5}{a_5c}.$$

So if a_5c were equal to 10 m. and cb_8 to 15 m.,

$$(8-x)\times 10=(x-5)\times 15,$$

or $$80+75=25x.$$

That is, $$x=\frac{155}{25}=6{\cdot}2\text{ m.}$$

If the *cotes* of A and B were m and n, a general expression for x, the *cote* of C, might be found in the same way.

Thus $$x=\frac{m\times cb+n\times ac}{ab} \text{ or } \frac{m\times CB+n\times AC}{AB}.$$

It will at once be seen how certain elementary geometrical surfaces are represented in this manner.

An inclined plane cut by equidistant horizontal planes presents straight horizontal sections which are projected as parallel and equidistant straight lines.

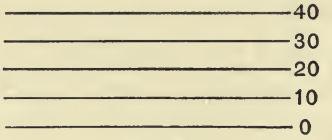


Fig. 36.

Two inclined planes that intersect each other so as to form a dihedral angle or book-shaped figure, when cut by equidistant horizontal planes, will present a series of similar angles. These angles are projected on the horizontal plane as corresponding angles, the apex of each lying in the projection of the line of intersection of the planes in question, and their *cotes* on either side of this line will be equal.

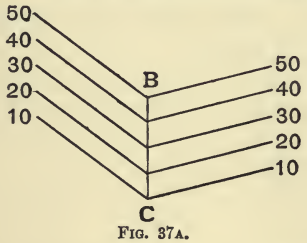


Fig. 37A.

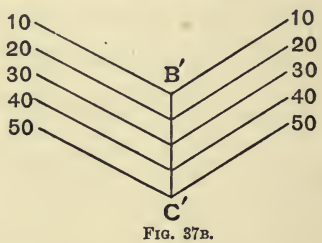


Fig. 37B.

According as the backbone (*arête*) of this figure is placed in front of or behind the observer, the *cotes* will ascend or descend in value.

In Fig. 37A, for example, the enveloping angle corresponds with the lowest section, while in Fig. 37B it corresponds with the highest.

In Fig. 37A, therefore, the two planes intersecting along *BC* suggest the primitive form of a croupe of which *BC* would be the ridge line. Whereas in Fig. 37B the planes that intersect along *B'C'* give the rough idea of a valley of which *B'C'* is the path of the watercourse.

This method of indicating a croupe and a valley being derived from the projection of a dihedral angle

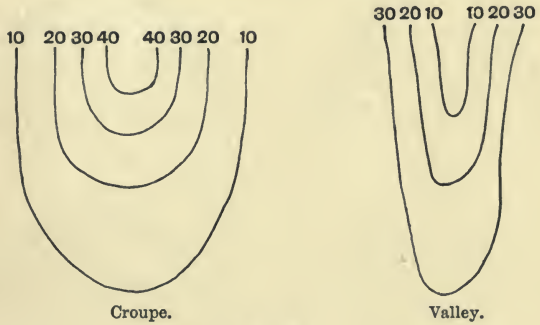


FIG. 38.

or book-shaped figure on the plane of reference, it follows that the representations of the croupe and the valley are theoretically similar in form, distinguished only by the *cotes* ascribed to their successive curves of intersection. The value of these *cotes* will be ascending in the first case and descending in the second.

A right cone with circular base, that is, the figure described by the revolution of a right-angled triangle, projected on the plane of the paper gives a series of concentric and equidistant circles, corresponding with the intersections of the intervening horizontal planes.

The intersections of a half-sphere with the same parallel and equidistant planes when projected horizontally also produce a series of concentric circles. These, however, rapidly become further apart as the height of the plane with which they correspond increases.

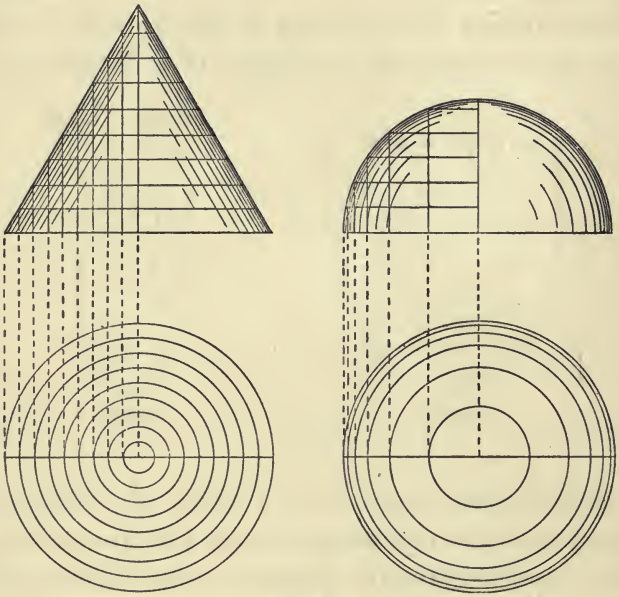


Fig. 39.

Hence it will be seen that the *mamelon* or knoll, which approaches the shape of a cone on its sides and that of a sphere at its summit, if cut by horizontal planes, say, five metres apart, will be represented by closed curves that are more or less nearly circular in form.

It must be observed that if the cone and the half-sphere were inverted they would give the same horizontal projections as before. Only, as was seen

for the dihedral angle, the order of the *cotes* of their contours would then be reversed; that is to say, they would diminish in value from the outside towards the centre.

As a hollow half-sphere resembles a basin, it follows that a basin-like depression in the ground will be represented by much the same figure as a

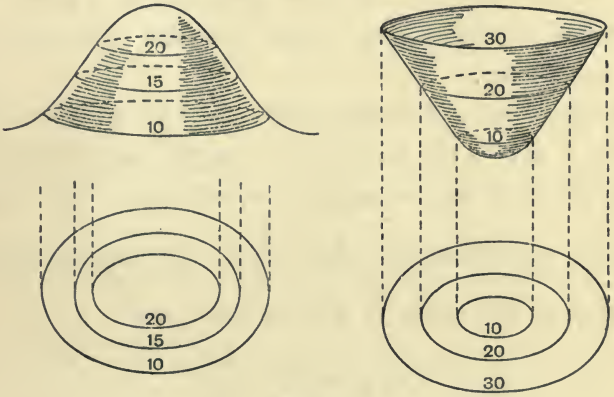


Fig. 40.

knoll, and will only be distinguishable from one by the descending progression of the *cotes* of its contours.

The system of equidistant horizontal sections will be seen then to satisfy three conditions essential to a faithful representation of the ground.

Firstly—Clearness of design, since from a single *cote* placed on one contour those of all the other contours in the same feature may be deduced.

Secondly—Easy comprehension of the form presented, since the projections that the features most often met with produce soon become familiar.

Thirdly—Sufficient exactness of outline, since the form of the ground can only be in doubt between two consecutive contours.*

EQUIDISTANCE.

The vertical interval that separates two adjacent horizontal sections of the ground is called the "natural equidistance," and remains constant for the same map.

Reduced to the scale of the map in question this factor is given the name of "graphic equidistance."

Thus, if E is the natural equidistance and e the graphic equidistance, while $\frac{1}{M}$ is the representative fraction of the scale, it will be seen that

$$e = \frac{1}{M} \times E = \frac{E}{M}.$$

And, if E', E'', etc., are the natural equidistances and $\frac{1}{M'}$, $\frac{1}{M''}$, etc., the representative fractions for other maps, then

$$e = \frac{E}{M} = \frac{E'}{M'} = \frac{E''}{M''}, \text{ etc.}$$

For the sake of uniformity and to simplify the execution and reading of the various maps, a constant

*There are, of course, various other ways of showing levels on the map. For instance, profiles may be set off along the roads. But that method, though admirably suited to a touring map, does not indicate the nature of the entire ground, as horizontal contours do. These or cognate means have therefore been almost exclusively used by the military cartographers.

value has been given to e for all scales, which is usually fixed at a quarter of a millimetre.*

Where e is equal to $\frac{0{\cdot}001 \text{ m.}}{4}$, and E consequently equal to $\frac{M}{4000}$, the natural equidistance:

In the scale of $\frac{1}{10{,}000}$ is 2·50 m.
In the scale of $\frac{1}{20{,}000}$ is 5·00 m.
In the scale of $\frac{1}{40{,}000}$ is 10·00 m.
In the scale of $\frac{1}{80{,}000}$ is 20·00 m.

And will be obtained for any scale by dividing its denominator by 4000.

One advantage of having a fixed value for e is that whatever the scale of the map may be the distance between two consecutive contours is the same for the same slopes.†

*In plans on a large scale or maps of mountainous country the graphic equidistance is sometimes made half a millimetre or even a whole millimetre. In certain cases also it is taken as a fifth of a millimetre, so as to make the real equidistance an integral number of metres.

†Take a distance AB on an incline of 1 in 12, for instance. In the scale of 1/40,000, the point B will be 10 metres higher than the

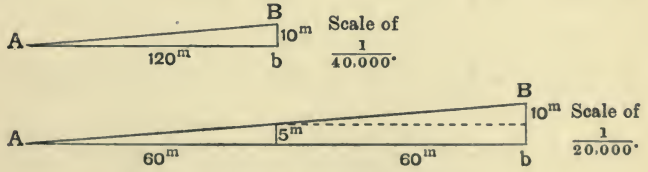


FIG. 41.—[Distance exaggerated tenfold.]

point A, and the projection of AB will denote twelve times this length. In the scale of 1/20,000, where the contour interval is 5 metres, Ab will still represent 120 metres though given twice the length it had before. Also Bb will still represent a height of 10 metres; but another contour will now intervene midway between A and B at a height

This was the case under the so-called "normal system," formerly employed by British map-makers. In the scale of twelve inches to the mile the contour interval then was 10 feet, in that of six inches to the mile 20 feet, and in that of three inches to the mile 40 feet. In all these scales the graphic equidistance was $\frac{1}{44}$th of an inch, or rather more than half a millimetre. And, as the scale became smaller, the vertical interval between the contours, or natural equidistance, was proportionately increased; that is to say, it was in inverse proportion to the size of the scale.

COMPARISON OF INCLINES.

The gradient of the line between two points on the ground is expressed by a fraction, the numerator of which is the difference of level of the two points or difference of their *cotes*, and the denominator is the length of the horizontal projection of the line.

Suppose the incline of the line *MN* on the ground to be contained between *m* and *n*, the points of intersection of its projection with two contours, the *cotes* of which are respectively 30 and 35.

Then the gradient of this incline will equal

$$\frac{\textit{cote of } M - \textit{cote of } N}{\text{projection of } MN}.$$

Now, *cote* of M − *cote* of $N = 35 - 30$, which is the same thing as the natural equidistance E; and

of 5 metres above *A*. The contours of this scale will, therefore, only intercept a length representing 60 metres, so that the distance between them for the same slope remains the same.

the projection of *MN* is in reality equal to the length of *mn* multiplied by the denominator of the scale adopted.

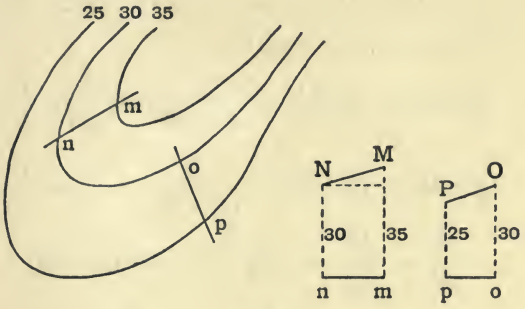


FIG. 42.

That is, the gradient of $MN = \frac{E}{mn \times M}$.

Similarly, if *OP* be another line,

$$\text{the gradient of } OP = \frac{E}{op \times M}.$$

$$\text{Therefore} \quad \frac{\text{gradient of } MN}{\text{gradient of } OP} = \frac{\frac{E}{mn \times M}}{\frac{E}{op \times M}}$$

$$= \frac{op}{mn}.$$

This is expressed by saying that: On a map in contours the gradient of a line on the ground is inversely proportional to the length of the projection of that line between two consecutive contours, reduced to the scale of the map.

Moreover, when the graphic equidistance remains constant it is possible to compare inclines on two maps of different scales.

In two maps, the scales of which are $\frac{1}{M}$ and $\frac{1}{M'}$, take the projections *ab* and *cd* between two consecutive contours that correspond with the lines *AB* and *CD* on the ground.

Then gradient of $AB = \frac{E}{ab \times M} = \frac{1}{ab} \times \frac{E}{M}$,

and gradient of $CD = \frac{E'}{cd \times M'} = \frac{1}{cd} \times \frac{E'}{M'}$.

But $\frac{E}{M} = \frac{E'}{M'} = e$ ($\frac{1}{4}$ of a millimetre).

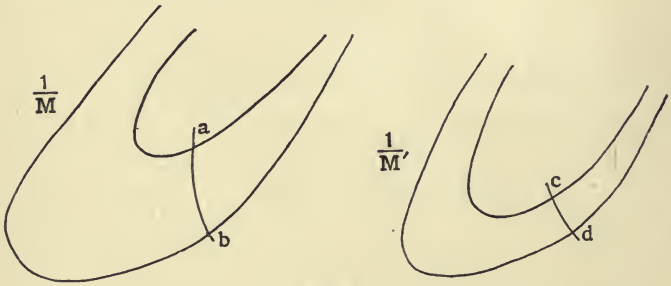


Fig. 43.

The two inclines may, therefore, be compared, thus:

$$\frac{\text{gradient of } AB}{\text{gradient of } CD} = \frac{\frac{1}{ab} \times e}{\frac{1}{cd} \times e} = \frac{cd}{ab}.$$

Like those of the same map they are inversely proportional to their projections.

If numerical values be assigned, say $ab = 0{\cdot}028$ m. and $cd = 0{\cdot}007$ m., then

$$\frac{\text{gradient of } AB}{\text{gradient of } CD} = \frac{0{\cdot}007 \text{ m.}}{0{\cdot}028 \text{ m.}} = \frac{1}{4},$$

or the incline of CD is four times as great as that of AB.

This may be verified if the gradients are ascertained when the maps are supposed to be respectively on the scale of 1/10,000 and 1/40,000. For then:

$$\text{gradient of } AB = \frac{\text{nat. equidistance of } 1/10{,}000}{ab \times 10{,}000}$$

$$= \frac{2{\cdot}50 \text{ m.}}{0{\cdot}028 \text{ m.} \times 10{,}000}$$

$$= \frac{250}{28 \times 1000} = \frac{1}{112},$$

and

$$\text{gradient of } CD = \frac{\text{nat. equidistance of } 1/40{,}000}{cd \times 40{,}000}$$

$$= \frac{10 \text{ m.}}{0{\cdot}007 \text{ m.} \times 40{,}000}$$

$$= \frac{10}{7 \times 40} = \frac{1}{28}.$$

$$\text{Therefore } \frac{\text{gradient of } AB}{\text{gradient of } CD} = \frac{\frac{1}{112}}{\frac{1}{28}} = \frac{1}{4},$$

which was shown to be the case when the size of the two scales was left out of consideration.

The inclination of the ground at any point is shown by the gradient of the line of greatest slope which passes through that point.

It has been proved already that the line of greatest slope of a plane is perpendicular to the intersection of the plane with the horizontal plane of projection. Similarly the line of greatest slope that passes through any point on the ground is perpendicular to the intersections of the ground with the two horizontal

planes situated immediately above and below the point in question. The projection of the line of greatest slope which passes through a point on the ground is consequently perpendicular to the contours traced on the map upon either side of the projection of that point.

A comparison on the same map, or on two maps of different scales, of the incline of the ground at two given points will then be made by comparing the gradients between two consecutive contours of the lines of greatest slope that pass through those points. This is to say, since the projection of each line of greatest slope within these limits is a measure of the separation of the contours, that the inclines of the ground at two points taken on the same map, or on two maps of different scales, are to one another as the distances between the contours nearest to these points.

REPRESENTATION OF THE GROUND BY MEANS OF CONTOURS.

(*a*) *The Mamelon.*—It has been observed that the *mamelon* or knoll resembles the half-sphere and is likewise represented by concentric, closed curves, the *cotes* of which, resulting from the heights of the intersections, increase from the outside or lowest contour towards the innermost or highest contour.

The difference of the *cotes* between each contour necessarily varies with the scale of the map. For, since the graphic equidistance remains constant, the

actual interval between the horizontal planes increases as the scale diminishes.

In the scale of $\frac{1}{10,000}$ this interval is 2·50 m., and in the scale of $\frac{1}{80,000}$ it is 20 metres. The *cotes* of the contours of a *mamelon* will, therefore, increase by 2·50 m. at a time, if the ground is represented on the scale of $\frac{1}{10,000}$, and by 20 metres at a time, if represented on that of $\frac{1}{80,000}$.

This fact should be noted, because it enables one immediately to discover the scale of a map from the *cotes* given to two consecutive contours.

(*b*) *The Croupe.*—The croupe is a convex feature, the shape of which is derived from that of a solid

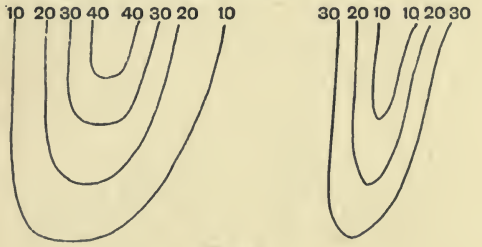


Fig. 44.

angle or dihedron. It is represented by concentric, open curves, the mouth turned towards the highest part of the ground, and the *cotes* of these contours progressively increase from the outside.

(*c*) *The Valley.*—The shape of the valley, also derived from that of the solid angle, is concave. Like the croupe it is represented by concentric, open curves, but their mouth is turned towards the lowest part of the ground, and the *cotes* of the contours increase from the inside.

Two croupes back to back form a *mamelon*.

Two valleys facing each other form a basin.

The *mamelon* may therefore also be derived from the four-sided pyramid, and the basin from the same figure inverted, in the same way as the croupe and the valley are derived from the solid angle.

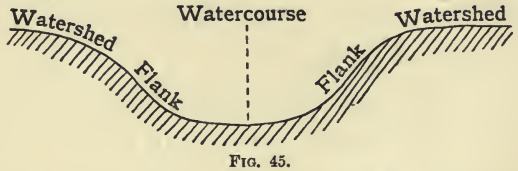
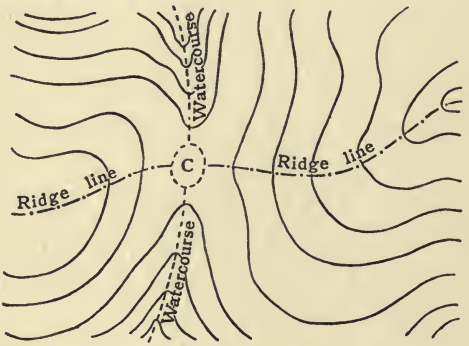


FIG. 45.

The line in a croupe that joins the contours where they are farthest apart is called the ridge line (*ligne de faîte*) or line of parting of the waters. The line that joins similar points in a valley is called the watercourse (*Thalweg*).



C. *indicates summit of col*

FIG. 46.

The slopes on either side of the ridge line are called watersheds (*versants*), those situated on either side of the watercourse constitute the flanks (*flancs* or *berges*) of the valley.

(*d*) *The Col.*—Two valleys back to back or two croupes facing each other form a col.

In order to understand the method of representing this formation, imagine two croupes *C* and *C′* placed opposite each other and near enough for one to impinge on the other.

The contours that represent them will intersect each other as indicated in the accompanying figure.

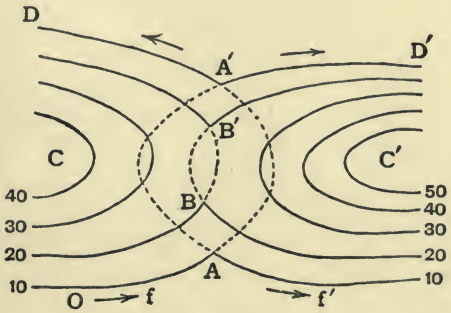


Fig. 47.

Now suppose an observer to move from the point *O* on the contour 10 in the direction of the arrow *f*. On reaching the point *A*, he will meet the contour 10 of the croupe *C′*. And, in order to continue to advance on the same horizontal level without tunnelling under the slope of that hill, he must follow contour 10 of the croupe *C′*. That is to say the part of contour 10 beyond the point *A* below croupe *C′* will be hidden, and should not, therefore, appear on the map. It is the same for the part of contour 10 beyond the point *A* that lies beneath croupe *C*.

If, however, one imagined that the observer, after reaching the point *A*, had travelled under

the surface of croupe C' on the same level as before, he would arrive at the point A'. There he emerges from the slope, and may continue to advance, without change of level, either in the direction of AD or AD', which portions of contour 10, not being mutually destructive, should appear on the map.

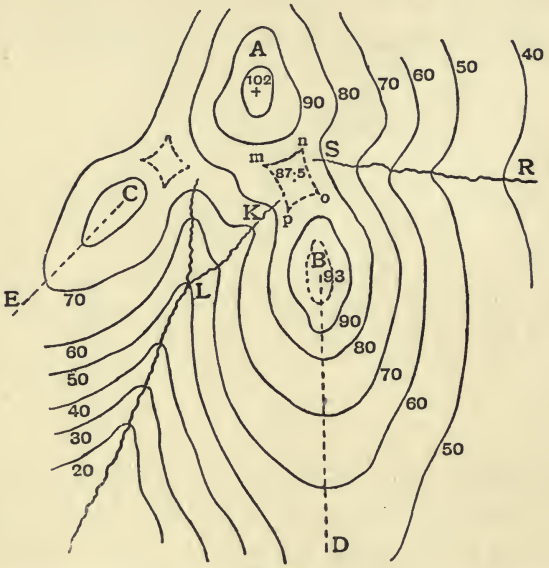


Fig. 48.

Representation in contours of three *mamelons A*, *B* and *C*; *RS* and *LK* valleys; *BD* and *CE* ridge lines.

For similar reasons the parts of contour 20 between B and B' are cancelled. But an observer may follow contour 30 of croupe C without meeting contour 30 of croupe C', and *vice versâ*. The whole of these contours should, therefore, be shown on the map; and, of course, those above them also.

Thus only the visible portions are taken into account; for the contours are supposed to be traced

on the surface of the ground and not beneath it. Consequently the col formed by the two croupes *C* and *C'*, placed opposite each other, should be represented as in Fig. 47 with the dotted lines left out.*

Similar conditions may be shown to exist when the col is considered as being formed by two valleys back to back; and generally speaking, both formations may be said to be present.

LINES OF GREATEST SLOPE.

Through each point on the surface of a croupe there passes a line of greatest slope, constituted by the incline of the ground at that point. The most important of all these lines is the "ridge line,"

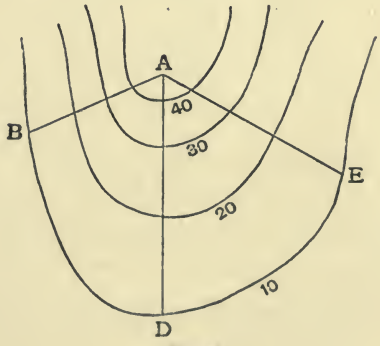


Fig. 49.

which may be regarded as being produced by the meeting of the two sides of the croupe. It has the smallest gradient as it corresponds with the biggest separation of the contours.

* It will be seen from the figure on page 82 that a col has ascents on two sides which are separated by descents on the other two sides.

The path of the watercourse is determined by the meeting of the two sides of the valley, and also corresponds with the biggest separation of the contours. Its gradient is, therefore, smaller than that of any other line of greatest slope in the same valley.

In order to trace a croupe with sufficient accuracy, it is enough to know the ridge line and two lines of greatest slope taken on opposite sides. One finds

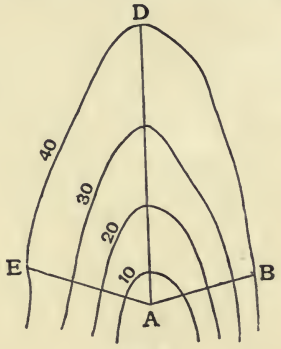


Fig. 50.

the points of passage of the contours on these three lines, and joins those of the same *cote*, giving the curves so traced the shape proper to the representation of a croupe.

A similar method is employed for representing a valley.

The contours, being *imaginary* lines, should not arrest attention more than is necessary to enable one to estimate the form of the ground with ease. They should, therefore, be marked with a thinner line than the real objects in the map, of which the planimetry consists.

Certain contours are reinforced sometimes to facilitate the reading of the map; every fourth or fifth, say, being traced with a thicker line. This is termed a master contour.

Additional contours are interpolated also when the gradient is very small to indicate the shape of the ground; but they should be traced with dotted lines in order to prevent any mistake as to the mean equidistance.

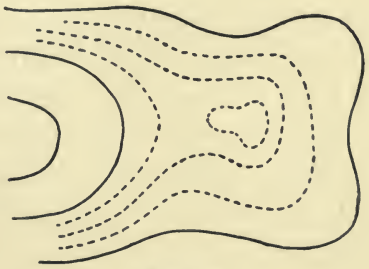


Fig. 51.

On the other hand, in very great inclines the contours would fall too near together. To avoid this, the master contours are first traced for this portion of the ground, and then the ordinary contours are put in, but broken where the incline is most violent. When the slope is over 45 degrees, or the gradient steeper than 1 in 1, the contours are replaced by the conventional sign for escarpments (rocks).

From a contoured map it is easy to make a plan in relief, or plastic model of the ground; and nothing gives one a better idea of the principle on which the map itself is constructed.

A tracing of the map is first made and pasted upon a flat board. Then directing lines are drawn across

the contoured parts. In these sections each contour with its line of direction is now traced on a separate piece of cardboard a quarter of a millimetre thick, that is, forty sheets to the centimetre of thickness. Cut out the lowest contour first and fix it in its place with small headless nails. Then cut out the others in order and place them according to their directing lines. The steps that result may now be transformed into continuous slopes by means of some plastic material, like putty. Then take a mould in plaster, which serves for making the plastic model or " relief plan." The final process is to paint in the planimetrical details ; but this cannot well be done until the cast has been coated with a mixture of zinc oxide, linseed oil, spirits of turpentine, and quick-drying varnish.

III.

MANNER OF SHADING.

Contours give a good general representation of the ground, but furnish no information as to any inequalities of the surface that may exist between two consecutive contours. This drawback is not great for large scale maps in which the natural equidistance is fairly small; but it is so for the small scale maps in which the natural equidistance is considerable. Besides, contoured maps have the more serious defect of not appealing quickly to the eye, and of consequently requiring a certain mental effort to be understood.

A method of representation has, therefore, been sought that would make the configuration of the ground apparent at first sight as in a photograph, while obeying certain rules in order that the design might not become vague.

HACHURES.

The appreciation of the form of an object results from the differences of light and shadow on its various sides. So an illusion of solidity may be procured by a suitable arrangement of outline and shading. The means of doing this in topography is called the *hachure*.

A hachure may be defined as a line of greatest slope for the ground between two adjacent contours. The hachures are therefore *normal* to the two contours that contain them.*

It follows that these two systems of lines—contours and hachures—are derived geometrically from each other, and that inclines may be compared by means of the hachures, since they are lines of greatest slope.

By breaking the hachures any inequality of the ground can be indicated, and by placing them nearer or farther apart a contrast of light and shadow will be obtained, which to some extent reproduces the relief of the various parts of the surface of the soil.

Now the more inclined the ground is to the horizontal plane the greater will be its area compared with that of its projection. Therefore, supposing it always to receive the same amount of light, it should be made darker in proportion to the angle of its inclination, being left white only when approximately horizontal.

This condition is obtained in practice by *la loi du quart*. Under that law the hachures are of a constant thickness and are separated from each other by a fourth of their length. The proximity of the hachures results, therefore, from that of their containing contours, or, what comes to the same thing, from the gradient of the incline.

*A line through any point of a curve is said to be normal to it when drawn at right angles to the tangent at that point.

Practically the hachures are traced according to *la loi du quart* in the following manner:

Take two contours *ab* and *cd* with a line of greatest slope *mn*, along which the first hachure is traced.

On *mb* a length *mm'* is set off equal to *mn*, and from the point *m'* the hachure *m'n'* is traced, along a line of greatest slope. The length *mm'* is then divided into four equal parts, and from each point of division a hachure is traced. The average distance

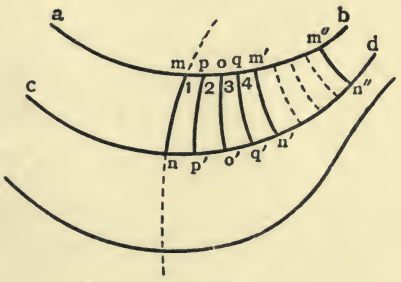


FIG. 52.

between them will be a quarter of their respective lengths, and *nn'* will also be divided into four approximately equal parts.

With a little practice, the division of each curvilinear rectangle, such as *mnn'm'*, is made by sight. The hachure *oo'* that divides it in two is first traced, and then the two other hachures *pp'* and *qq'* that subdivide it into four parts. And so on for the rectangle *m'n'n''m''*, etc.

The hachures, which vary in form with that of the containing contours, should be firmly traced and should end exactly at the two contours, to which they are normal.

Between parallel straight lines they are straight, parallel and equidistant.

Fig. 53.

Between parallel curves they are convergent.

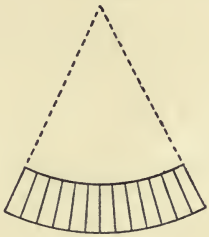


Fig. 54.

And between lines that tend to meet they are concave, opposite the point in which the contours would meet, if produced.

Fig. 55.

Between the same contours two hachures of opposite convexity should be separated by one straight hachure at the point where the contours are closest to one another.

To make the work easier as well as to give the desired accuracy, one begins by tracing in pencil on the contoured map a certain number of lines of greatest slope cutting all the contours. They should be sufficiently near to each other to leave no doubt

as to the direction of the hachures; but one is not absolutely obliged to trace a hachure on each of them.

To facilitate the re-discovery of the contours, as well as to make the map more agreeable to the eye, the hachures should not be drawn so as to continue each other.

There are certain exceptions to *la loi du quart.*

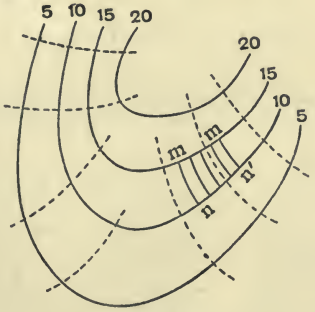


Fig. 56.

When the gradient of the ground is steeper than 1 in 8, that is to say, when the distance between the contours is less than 2 millimetres, the hachures are still drawn half a millimetre apart, but are made thicker the closer the contours come together.

When a gradient of 1 in 1 is reached, that is, when the contours are a quarter of a millimetre or less apart, they are discontinued and the hachures also are replaced by the conventional sign for escarpments or rocks.

The less the gradient the farther apart are the contours, so that the length of the hachures becomes considerable.

In the earlier sheets of the General Staff map the hachures were no longer traced when the incline fell below 1/64, but in later sheets the smallest incline represented by hachures is 1/144.*

In these inclines the hachures would be respectively 4 mm. and 9 mm. apart, so that there is already some risk of their being confused with the planimetrical details.

In order to secure uniformity in the spacing of the hachures a gauge has been made for each scale.

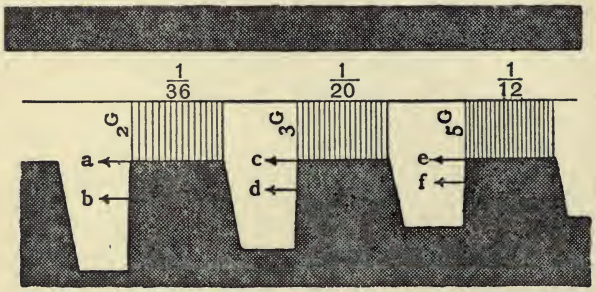


Fig. 57.

In these instruments, called *diapasons*, the ratio of black to white is equal to the value of the incline (gradient) multiplied by $\frac{3}{2}$.

The *diapason* is a sheet of paper indented as shown in the figure above and fixed on a thin strip of cardboard.

* The distance apart of the contours, or, what is the same, the length of the hachures for this incline, may readily be calculated.

If x denote the length of the hachure, e being the constant graphic equidistance, then

$$\frac{e}{x}=\frac{1}{144}.$$

That is,

$$x=\frac{0{\cdot}001 \text{ m.}}{4}\times 144$$

$$=0{\cdot}036 \text{ m., or } 36 \text{ mm.}$$

Above each opening the type of hachures proper to the various inclines $\frac{1}{36}$, $\frac{1}{20}$, $\frac{1}{12}$, etc., is placed. The distance of the contours apart for each incline is marked on the side of the teeth between the guiding points *ab*, *cd*, *ef*, etc.

To use the *diapason* one moves it over the design until the contours, between which hachures are

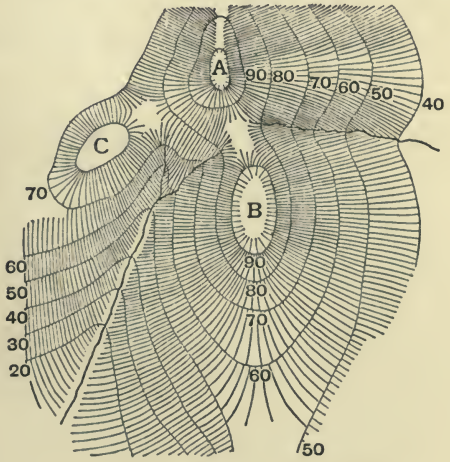


FIG. 58.

Representation in hachures of the ground delineated in contours in the figure on page 84.

to be drawn, correspond with the width of a pair of guiding points, *ab*, *cd*, *ef*, as the case may be.

It only then remains to prolong the hachures traced on the gauge above and give them the same thickness. To ensure accuracy, it is essential to consult this instrument unceasingly, and thus make quite certain that the hachures one traces are indeed those which correspond with the separation of the contours in the design.

REPRESENTATION OF ELEMENTARY FEATURES OF THE GROUND.

Croupe.—The croupes are represented by hachures that open out like a fan, and the smaller the incline becomes the farther they are apart. In practice one begins by tracing the hachures from the highest points and goes on downwards to the last contour, always remembering to draw the hachures that

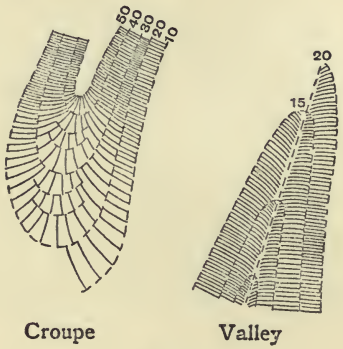


FIG. 59.

begin and end this feature of the ground to a fine point.

The ridge line is left free of hachures.

Valley.—The valleys are also represented by fan-shaped hachures, that one also begins to draw from the highest points, but those on either side are divided by the open space that is left along the watercourse, towards which they end in a fine point.

Mamelon.—The representation of a *mamelon* is similar to that of two croupes back to back. The

hachures spread out in fan shape round the blank space which constitutes the more or less flat summit of this feature.

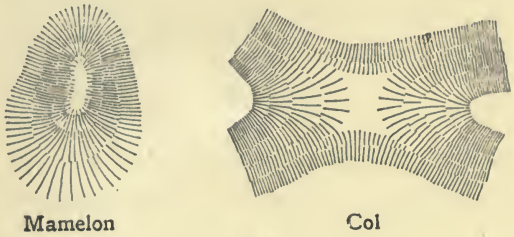


Fig. 60.

Col.—The cols are represented by two croupes face to face, the watersheds of which become the flanks of valleys.

Hachures are not traced in the central portion of a col.

An ingenious use of hachures is made in the larger scale maps to indicate the nature of roads.

Fig. 61.

A cutting or *route en déblai* is shown by hachures that become lighter as they approach the road.

Fig. 62.

An embankment or *route en remblai* is shown by hachures that become lighter as they leave the road.

A road cut out on one side and embanked on the other is consequently shown thus:

Fig. 63.

And a road cut *en corniche* in the face of a steep hill with a cliff on the outside is represented thus:

Conventional sign for escarpments.
Fig. 64.

In the General Staff map on the scale of 1/80,000 the configuration of the ground has not been reproduced by an application of *la loi du quart*. The hachures are, indeed, traced along the lines of greatest slope, but their length is no longer in strict proportion to the distances that separate the contours. The object has been, with the aid of the *diapason*, to obtain a general representation of the levels, bringing into relief the characteristic features of the ground, but at a sacrifice of form as well as of certain details.

One cannot reconstitute the contours of this map by joining the feet of each rank of hachures.

Nevertheless, students of topography are expected in France to be able to enlarge any given portion of the Staff Map on the scale of 1/80,000 into a contoured map on the scale of 1/40,000.

Among various kinds of hachures used in other

countries, the German "hachures Müffling" may be cited as instances of an almost perverse ingenuity. For they not only give the map a darker tint as the slope increases, but vary in design every five degrees, so that the gradient becomes apparent at first sight.

CONSTRUCTION OF A PROFILE.

Every vertical plane cuts the ground along a curving line that is called a profile. To depict the form of the surface along any given line, therefore, the ground is imagined to be cut by a vertical plane. Its profile may then be constructed on geometrical principles.

Take a portion of the ground between the contours 30 and 60, and let *xy* represent the line traced by its vertical section.

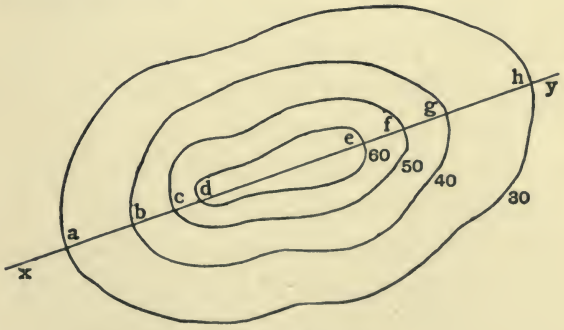


FIG. 65A.

The vertical plane first cuts the highest contour at two points *D* and *E*, which are projected at *d* and *e*, then the second highest contour at two points *C* and *F*, projected at *c* and *f*, the third contour

next at two points *B* and *G*, projected at *b* and *g*, and finally the lowest contour at two points *A* and *H*, which are projected at *a* and *h*.

If these points on the ground, *D* and *E*, *C* and *F*, *B* and *G*, *A* and *H*, are supposed to be joined in the vertical plane that contains them, it is obvious that the lines *DE*, *CF*, *BG*, and *AH* will be parallel, since they all lie in horizontal planes. They will also be equidistant, as the vertical interval between each pair of them is the same—in this case 10 metres.

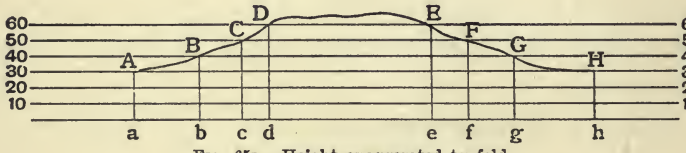


FIG. 65B.—Height exaggerated tenfold.

Parallels are consequently drawn to a horizontal straight line, so as to represent the height of the contours from 10 to 60 metres, and on this base line the distances along *xy* intercepted by the points *a*, *b*, *c*, *d*, and *e*, *f*, *g*, *h* are set off.

Then, if perpendiculars are drawn from *a* and *h* to the parallel marked 30, the relative positions of *A* and *H* will be indicated. So on, when perpendiculars are raised at *b* and *g* to the parallel marked 40, from *c* and *f* to that marked 50, and from *d* and *e* to that marked 60, those of B, G; C, F; D, E.

The points *A*, *B*, *C*, *D*, *E*, *F*, *G*, *H* may now be joined with a symmetrical curve, suggested by the lateral spacing of the contours, so as to show the profile of the ground along the line *xy*.

Owing, however, to the impossibility of drawing parallels at the true graphic equidistance of a quarter of a millimetre apart, only a comparative representation of the form of the ground has been obtained. The element of height, at any rate in the smaller scales, has to be exaggerated, so that one gets an exalted profile. In the accompanying figure the height has been made ten times as great as the natural equidistance of 10 metres warrants. But had the scale been 1/10,000 instead of 1/40,000, then only every fourth contour of the map would be indicated in the figure, and each ten metres would properly be represented by a space of one millimetre. When the graphic equidistance is exaggerated four times, therefore, the profile so drawn may be described as having been raised to the scale of 1/10,000 from that of 1/40,000.

If a faithful reproduction of the slopes indicated in the figure were required, the distances *ab*, *bc*, *cd*, etc., would have to be multiplied ten times before being set off along the base line. In this manner an approximately true picture of the profile would be obtained on the enlarged scale of 1/4000.

Only by trial, and not then without difficulty, can profiles be constructed from the General Staff map of 1/80,000, in which the hachures are drawn by means of the *diapason*; but these appeal so directly to the eye that they enable one with a little practice approximately to determine the form of the ground between any two given points.

It may sometimes be necessary to ascertain the

cote of a point lying between two contours; and it can be done in the following way:

Take the *cote* of a point *C* between the contours 10 and 20 of a map on the scale of 1/40,000.

Through the point of projection *c* draw any line *ab* that cuts the contours, and measure it, also the length of *bc*.

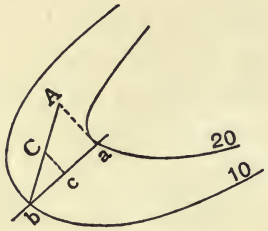


Fig. 66.

Draw *aA* perpendicular to the plane of contour 10 and equal to the difference in height of contours. The incline being considered uniform, *bA* will be a straight line which the perpendicular to plane of contour 10 from *c* will meet at *C*. The triangles *Aba* and *Cbc* then are similar.

Therefore $$\frac{Aa}{Cc} = \frac{ab}{bc},$$

or $$Cc = \frac{Aa \times bc}{ab}.$$

So, if one supposes that *ab* measured 10 mm. and *bc* 4 mm.,

$$Cc = 10 \text{ m.} \times \frac{4}{10} = 4 \text{ m.}$$

The *cote* of *C* is consequently 14 metres.

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Scale of $\frac{1}{80,000}$.

Facing p. 103

IV.

FRENCH GENERAL STAFF MAP.

THE map of France made by the General Staff took the place of Cassini's map, which was begun in 1733 and not finished till 1815.*

To Cassini is due the first topographical map of a country, the survey for which was based upon the careful measurement of a meridian arc and upon the most accurate geodetic observations possible. The scale of his map would be represented by the fraction $\frac{1}{86,400}$. It consisted of 184 sheets, engraved on copper.

The General Staff map was initiated by Napoleon I., who caused its method of construction to be studied; but its execution was only begun in 1818.

Starting from the meridian of Paris and Barcelona, which was measured by Delambre and Méchain (1792-1799), for the purpose of establishing the metric system, the engineer officers were ordered first to construct by a scrupulously exact process several great chains of triangles, dividing France in opposite directions.

* The following particulars are chiefly taken from Major Espérandieu's *Guide Pratique pour la lecture et l'emploi de la Carte de L'État-Major* (9th edition), 1915.

These triangles were laid out along the meridians of Melun, Bayeux, Sedan, and Strasburg, afterwards following the parallels of Amiens, Paris, Bourges, Clermont-Ferrand, Rodez and the Pyrenees. Seven bases of verification were measured at Melun, Perpignan, Ensisheim, Brest, Bordeaux, Gourbeira and Aix, which direct measurements proved almost exactly equal to those obtained by calculation.

Thus the greatest difference discovered was in the Bordeaux base, and did not exceed 0·57 m. in a length of 14,119·65 m., which translated to the scale of 1/40,000 represents an error $\frac{1}{10}$ of a millimetre, and is, therefore, an altogether negligible quantity.

The chains of triangles with sides of from 40 to 60 kilometres, which were produced along six parallels and four meridians, formed a network, the meshes of which were filled by a subsidiary triangulation.

Each triangle was in turn broken up into smaller triangles by determining various points of the 2nd order, and from the stations so made the engineers, at the same time, fixed the position of a vast number of intermediate points of the 3rd order, destined to serve as starting-places for the purely local survey.

The geodetic and topographical operations began simultaneously. The triangulation of points of the 1st and 2nd order was finished in 1854 and of those of the 3rd order in 1863. The topographical surveys were completed in 1864.

To facilitate the work, the planimetry was made with the help of the cadastral* survey, the sheets of which, generally on the scale of 1/10,000, were first reduced to that of 1/40,000, adopted by the Ministry of War after a series of tests.

The first sheet of the General Staff map was printed in 1833 and the last or 273rd in 1882. The entire

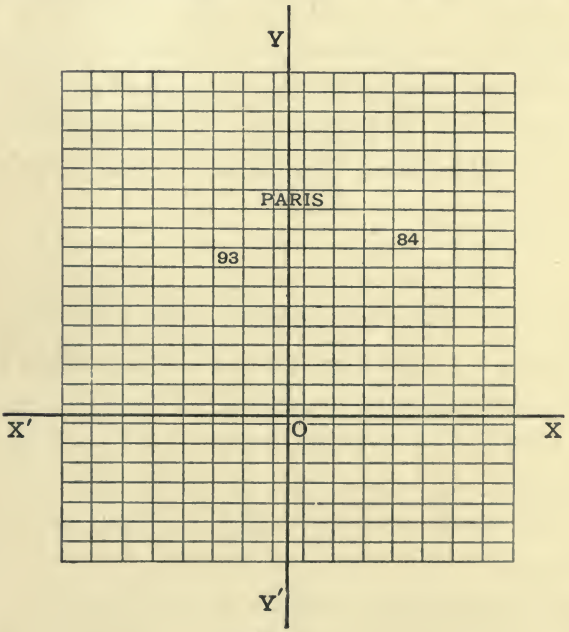

Fig. 67.

map cost almost a million pounds, and its most striking feature, considering the fact that nearly 800 soldiers and artists were employed on it, is the perfectly homogeneous nature of all its parts. For although the sheets were engraved by 65 different draughtsmen, every one of them appears to be by

* So named because it was originally made for revenue purposes.

the same hand. Placed together they cover a surface of over 160 square metres—13·20 m. in width and 12·30 m. in height.

The sheets were co-ordinated thus:

Two rectangular axes were taken, of which the vertical axis YY' was the meridian through the observatory of Paris, the horizontal axis XX' being the tangent to the mean parallel of latitude 45° N. These axes divide France into four parts: the North-East, North-West, South-East, and South-West. And each of these districts has been subdivided into rectangles that form the separate sheets of the map (Fig. 67).

INFORMATION GIVEN ON THE MARGIN.

Measured inside the frame that surrounds it each sheet of the General Staff map is 0·50 m. in height and 0·80 m. in breadth. The corresponding strip of country is, therefore, 40 kilometres wide and 64 kilometres long, covering an area of 256,000 hectares.

Each sheet bears a name and a number.

The name is written in the middle of the top margin in upright capitals 9 millimetres high. It is that of the most important place shown on the sheet, and not, as one might suppose, that of the place which occupies a central position in the drawing. (For instance, Le Mans.)

The number is written above the right-hand top corner of the frame in figures 8 millimetres high.

It is surrounded by a small rectangle, on the sides of which are other figures, serving to co-ordinate the sheet with respect to the perpendicular axes that intersect each other near Aurillac. (In the

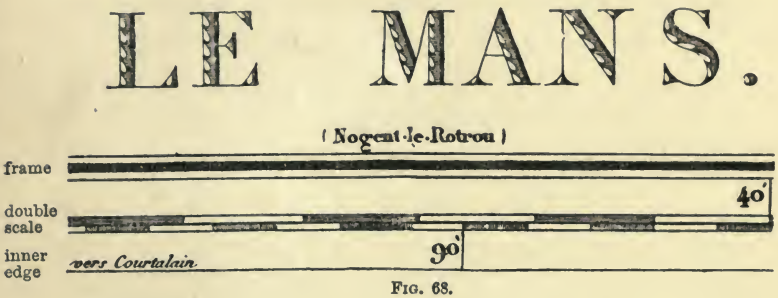

Fig. 68.

sheet of Le Mans the number is 93, having the figure 8 to the right and the figure 2 below.)

In the particular instance chosen, the figure 2 indicates that the sheet of Le Mans is in the second vertical column to the left of that which contains the sheet of Paris, and the figure 8 indicates that it

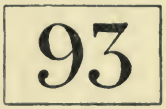

Fig. 69.

is in the eighth horizontal row above that which contains the sheet of Aurillac.

The distance in metres of each corner of the sheet from the axes is given inside the frame. That of the most distant corner may be found thus: $x \times 64{,}000 + 32{,}000$, and $y \times 40{,}000 + 20{,}000$, where, in this case, $x = 2$ and $y = 8$. Distance of left-hand

top corner of sheet 93 from the origin is therefore 160,000 metres W. and 340,000 metres N.*

The sheet of Aurillac, at the centre of which lies the intersection of the two axes, is co-ordinated thus:

Fig. 71.

Two small rectangles are to be found at the left-hand top corner of each sheet. The one, divided into nine equal parts, contains as many different numbers. In the case taken, the number 93 inscribed in the

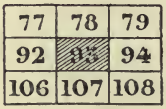

Fig. 72.

division covered with hachures, is the index number of the sheet of Le Mans. The other numbers, 77, 78, 79,

* x is either E. or W. and y is N. or S., as shown by position of small figures outside the rectangle. For example, in the sheet of Mirecourt they are given thus:

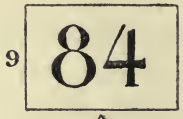

Fig. 70.

that is, the right-hand top corner being furthest from the axes its distance is measured to the East and North. To the total breadth and height of the intervening sheets must always be added half a corresponding side of the sheet of Aurillac.

92, 94, etc., indicate those of the neighbouring sheets.

This information is useful, as it enables one to find quickly the number of the sheet that prolongs the details of planimetry or elevation interrupted by the frame of the map one is looking at.

The letters *a*, *b*, *c*, *d*, *e*, etc., in the second rectangle shown below, indicate the different parts of the sheet surveyed by each of the

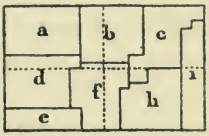

Fig. 73.

officers engaged in its construction. The names and rank of these officers, together with the date of their operations on the ground, are placed on the right of this second rectangle.

In the lower margin is a double scale expressing on the one side distance in thousands of metres and on the other in kilometres.

The names of the towns printed in small letters on each margin along the frame, *Nogent-Le-Rotrou*, for instance, are those of the sheets that touch the one under consideration.

At the right-hand corner of the lower

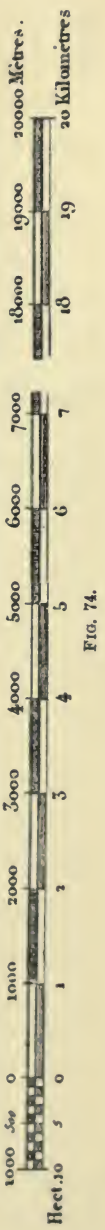

Fig. 74.

margin the names of the engravers are placed, in this case, thus :

*Gravée : le trait par Thullier, la lettre par Hacq, le figuré du terrain par Dandeleux.**

At the left-hand corner the dates of first publication and latest revision are given, in this case, thus :

Levée par les Officiers du Corps d'État-Major, et publiée par le Dépôt de la Guerre en 1846.—Révisée en 1906.

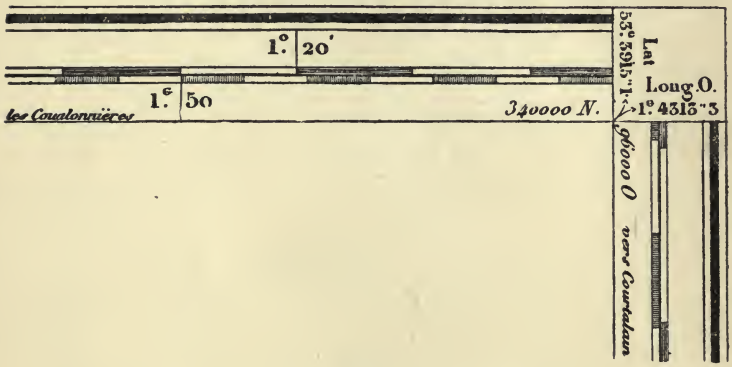

Fig. 75.

Each sheet of the General Staff map is surrounded with a triple frame. The exterior edge formed by a broad line between two fine lines is merely ornamental. The frame between it and the straight edge of the map, which somewhat resembles the metrical scale in the lower margin, is in fact a double scale that enables one to ascertain the longitude and latitude of any point in the map. The inner

* Everything in the map that properly belongs to the planimetry is called *trait*, the written portion takes the name of *lettre*, and the representation of form is termed *figuré du terrain*.

divisions of this scale are expressed in grades and the outer divisions in degrees. Both of them are marked at intervals of 10 minutes.

The double scale and the exterior frame are broken at the four corners in order to admit a little square as shown in the figure above. The longitude and latitude of each corner of the particular sheet are indicated in these little squares. In this case the top right-hand corner of sheet 93 is taken, which is shown to be $1^{G}\cdot4313''3$ Long. West of the meridian of Paris and $53^{G}\cdot3915''1$ Lat. above the equator, it being unnecessary to mark North. This corner is obviously 64,000 metres nearer to the meridian of Paris than the top left-hand corner, the distance of which was found by means of its numerical co-ordinates.

FURTHER PARTICULARS.

The General Staff map, having been drawn by Bonne's modified conical method, described in the introduction, only the meridian of Paris, longitude 0, is a straight line parallel to the lesser sides of the frame. The other meridians are curves, approximating to circles of great radius which have not the same centre. The parallels, however, are circular arcs that have a common centre situated on the meridian of Paris.

For various reasons and especially for convenience of handling, the sheets are now issued in four sections, on each of which the metrical scale is given. These

quarter sheets belong to the "Type 1889," and are published in a zincographic reproduction at thirty centimes a piece or engraved at one franc.

The admirable provisional edition, which is now being issued by a rapid process, is not only beautifully clear but brings the revision of the map up to 1913.

It should be remembered that the distance measured on the map is proportionately equal to that which one would have to cover on the ground only when the country is quite flat. In order to take into account the inequalities of the ground, it has been found by experience that one must add about one-third to distances measured on the map over strongly marked inclines.*

* A rough idea of distance on the map may be obtained with a five-centime piece or *sou*, the diameter of which is exactly $2\frac{1}{2}$ cm., so that in the scale of 1/80,000 it covers an area 2 kilometres wide.

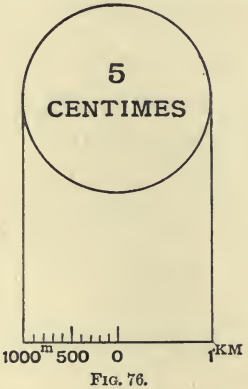

FIG. 76.

Indeed, an approximate scale may be made with one of these coins, thus: Draw a circle very close to the rim. Then drop tangents from opposite ends of a diameter on a parallel straight line outside the circle and bisect it.

A comparative scale of paces to metres can be made, on the assumption that the average man covers a distance of 100 metres in 130 paces, so that a thousand paces represents about 770 metres. The usual method employed is to measure 7·7 cm. along a straight line from a point marked *o*, to represent both 1000 paces and 770 metres, which distances are marked off above one another, also

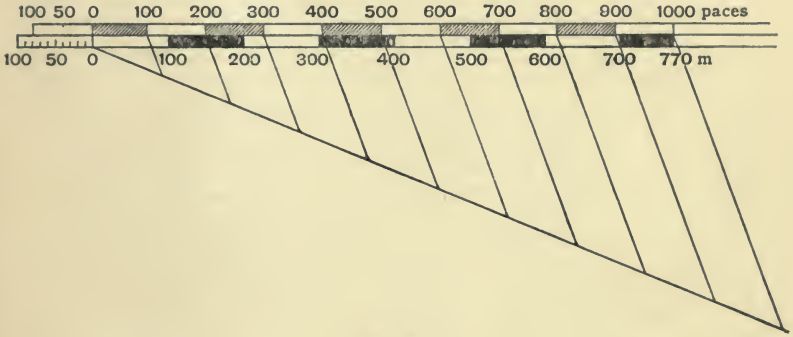

Fig. 77.

the intervening centimetres that each represent 100 metres.

Then on an oblique line through *o* ten equal sections are taken and graduating parallels drawn that divide the part of the original line between the point marked 770 m. and *o* into tenths, each of which will represent 100 paces.

The General Staff map is not coloured, but an artistic effect of mezzotint is given by the hachuring in the sheets that represent mountainous districts, like the Vosges.

The extension of the General Staff map to the scale of 1/50,000, a second edition of which is now in the

press, has been made by photography. Its detail does not differ, therefore, from that of the original, except in size.

There is also a new map in preparation by the *Service géographique de l'Armée* on the scale of 1/50,000. Hill features are shown by shading and by contours at 10 metre intervals, the graphic equidistance being one-fifth of a millimetre. This is said to be one of the finest topographical maps in existence.

V.

ORIENTATION.

Literally the word means ascertaining one's direction with respect to the East; and it is applied in this sense to churches, the high altars of which in Europe are usually so placed that when turning towards them one faces the East. The orientation of a church may, therefore, afford information in a strange country. But it is the line from North to South, given by the meridians of the map, that one wants to establish in the field. This is, of course, most readily discovered with the compass, the direction of the magnetic needle being in France about 14° to the West of the true North, sometimes indicated there by the letters N_V.*

FIG. 78.

The South may be found when the sun is up by pointing the hour hand of a watch, held horizontally, towards the sun, for the diameter of the dial that bisects the distance between the figure XII and the hour hand will then give the direction of the South. Thus, at four o'clock the figure II will

* This deviation is decreasing by about 10′ every year.

approximately point South and the opposite figure VIII North.

This depends upon the fact that at twelve o'clock, if the watch be set to the hour of the place, the sun will be due South. Now, if the figure XII is supposed to be turned towards the sun at noon and the watch is left undisturbed on a table till four o'clock, the hour hand will be found to have travelled twice as

Fig. 79.

fast as the sun. For it makes a complete revolution in twelve hours, while the sun, with apparently the same circular movement from East to West, takes twenty-four hours to return to its starting-place. Consequently the distance of the sun from the South at 4 p.m. may be measured by half the arc of the dial between the figures IIII and XII.

The watch must, therefore, now be turned so far round from right to left, that is, in a contrary direction to the movement of the hands, in order that the

figure II may point Southwards. But the figure IIII, having been moved through a precisely equal arc, will then occupy the previous position of the figure II and be pointed at the sun, which is what was originally proposed.

On the opposite side of the dial between 6 a.m. and noon the sun appears in advance of the hour hand which is in its second revolution. But the sun will be in the South again when the hour hand reaches the figure XII, and the same law applies as before. Thus, if at, say, eight o'clock one turns the hour hand to the sun, the diameter of the dial through the figure X will point to the South.

In that permanently fixed watch the sundial, the point of noon is placed due North, so that the shadow cast by the narrow edge of the stile or gnomon may fall on it when the sun is in the South.

The stile, which is fixed in the plane of the meridian, is set so as to make an angle with the horizontal plane equal to the latitude of the place, and is therefore parallel to the earth's axis—approximately the stile may be said to point to the pole star.

The instant at which the sun crosses the meridian is called *apparent* or *true* noon. The angle which the plane containing the sun and the stile makes with the meridian plane is called the *hour angle* of the sun. Since the hour angle increases from 0° to 360° in the interval between two successive transits of the sun across the meridian, that is, in 24 hours, the hour angle increases by 15° each hour. Hence the plate of a sundial may be graduated, whether it

be horizontal, or vertical, or inclined at any angle to the horizontal, by taking planes through the stile making angles 15°, 30°, etc., with the plane of the meridian.

A sundial may be easily constructed in any plane by the following method. Let a skeleton sphere

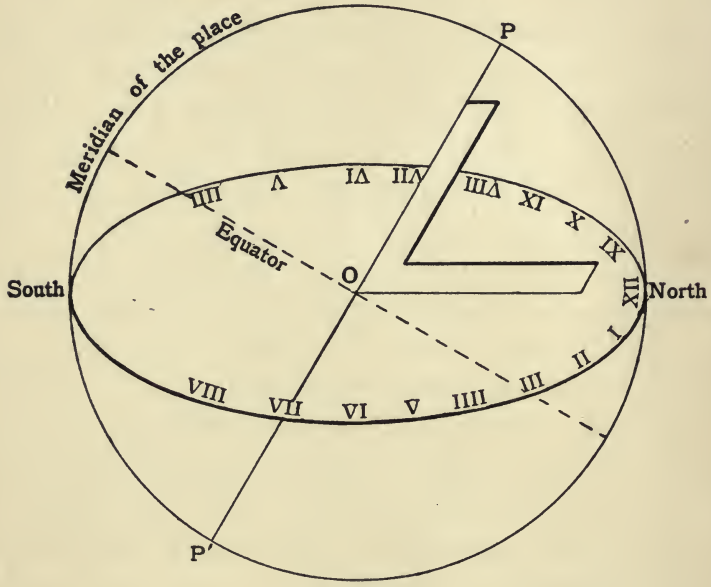

Fig. 80.

be constructed of twelve equal circles made of fine stiff wire, the planes of the circles intersecting in a common diameter, and each pair of successive planes inclined at an angle of 15°.

Let PP' denote the common diameter of these circular planes; then to graduate a sundial at any place, PP' must be fixed so as to be parallel to the polar axis of the earth; and the points in which

the successive wires cut any given plane through the centre of the sphere will give the graduations for a sundial, the plate of which is in that plane.

Figure 81 shows how the graduations would be obtained for a sundial, the plate of which is horizontal.

It must be remembered that a sundial gives the apparent *solar time*.

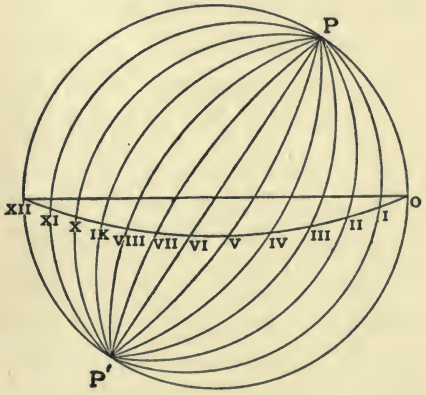

Fig. 81.

On comparing the time as given by a sundial, which has been accurately graduated, with an ordinary clock on different days of the year, it will soon be found that there is often a considerable disagreement.

The time as given by the sundial may be as much as 16 minutes fast or slow compared with the clock. The reason for this is that the interval of time between two successive transits of the sun across the meridian is not constant, but varies from day to day, being sometimes half a minute more or half a minute less than the average.

Apparent or true time is not, therefore, strictly speaking a measure of time, since the essence of all measurement is an invariable unit or standard. It is the average interval between two successive transits of the sun across the meridian that is taken as the ordinary standard of time, or, as it is called, *mean* time.*

The difference between mean time as given by a clock and apparent solar time as given by a sundial is called the *equation* of time. In some almanacks the equation of time is given for every day of the year; if it is positive, the equation of time must be added to the dial time to obtain the clock time; and if it is negative, the equation of time must be subtracted from the dial time to obtain the clock time. Some almanacks state whether the sun is *before* or *after* the clock: in the former case the equation of time must be subtracted from, in the latter case added to, the dial time to obtain the mean time.

If the equation of time cannot be found from an almanack, it may be very easily determined on any

* Captain E. de Larminat explains the way in which mean solar time has been arrived at, as follows:

"An ideal sun has been imagined, that would have a uniform motion and travel round the celestial equator through equal arcs in equal spaces of time, while making the same number of apparent revolutions about the earth in the year as the real sun. The time given at each moment by the hour angle of this ideal sun, called *mean* time, is characterised by the equality of its days, hours, minutes and seconds.

"Mean time is consequently a proper measure of time, corresponding to our wants. But as the *mean* sun is not to be observed, it has been necessary to calculate for every day of the year the difference between the hour given by the true sun and that which would be given by its invisible rival."

day when the time of sunrise and the time of sunset are known. Thus let r and s denote the times of sunrise and sunset, and let E denote the equation of time. Then since the interval from sunrise to apparent noon must be equal to the interval from apparent noon to sunset,

$$12 - r + E = s - E,$$

and therefore

$$E = \tfrac{1}{2}(r + s - 12).$$

Thus, if the time of sunrise is 7.40 and the time of sunset is 4.48,

$$E = \tfrac{1}{2}(7\text{ h. }40\text{ m.} + 4\text{ h. }48\text{ m.} - 12\text{ h.})$$
$$= 14 \text{ minutes.}$$

The equation of time is chiefly due to two causes: (1) the motion of the earth in its elliptical orbit about the sun is not uniform; (2) the plane of the earth's orbit is inclined at an angle to the plane of the earth's equator.

The changes due to these two causes are shown in Figure 82 by dotted curves; while the changes in the value of the equation of time are shown by the dark-lined curve.

If the equation of time is denoted by E, and if the parts of E due to these causes are denoted by E_1 and E_2, so that $E = E_1 + E_2$, it will be seen from this figure that E_1 vanishes on January 1st and on July 1st; E_1 is positive during the first half of the year and negative during the second half; and it attains its greatest value of 7 minutes (approximately) midway between these dates. On the other hand, E_2 vanishes four times a year, namely, at the

equinoxes and at the solstices, attaining its greatest value of 10 minutes (approximately) midway between each of these days.

E vanishes four times a year, on or about April 17th, June 15th, September 2nd, and December 26th. *E* is positive from December 26th to April 17th, attaining its greatest value of about 14 minutes

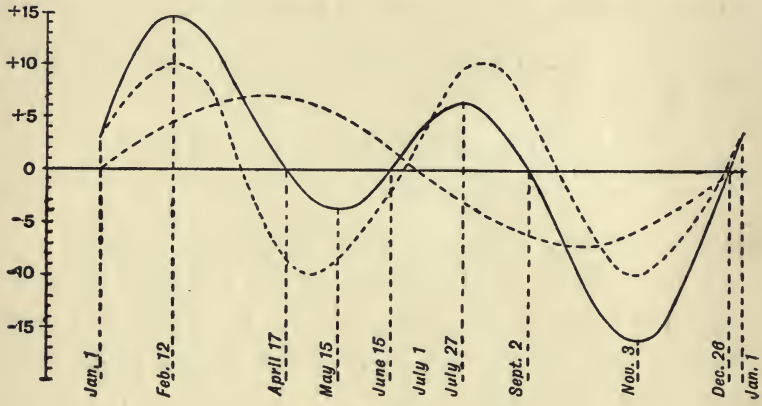

Fig. 82.

25 seconds on February 12th; it is negative from April 17th to June 15th, attaining its greatest value of 3 minutes 47 seconds on May 15th; it is positive from June 15th to September 2nd, attaining its greatest value of 6 minutes 20 seconds on July 27th; and it is negative from September 2nd to December 26th, attaining its greatest value of 16 minutes 22 seconds on November 3rd.

At night, if the sky be clear, the direction of the North may be found from the pole star or from the position of the moon when visible.

Everybody knows the Great Bear or "Dipper,"

as the Americans call it, always to be seen in the northern hemisphere.

The two brightest stars in this constellation, α and β, commonly known as the "pointers," give a line that, when produced by about five times the length between them, reaches the bright star in the tail of the Little Bear, called α Polaris, which is at present within a degree and a half of the pole.

Anyone who was in the habit of looking for the Great Bear about the same time every night would

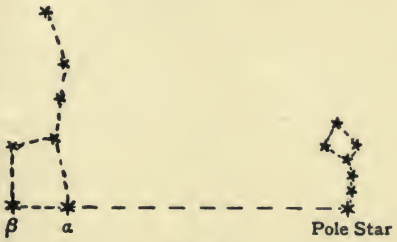

FIG. 83.

observe that it seemed to make a complete circle round the pole star during the year. But if he were to watch it at intervals for several hours on any night he would perceive a much more rapid movement. Indeed, owing to the rotation of the earth on its axis, the Great Bear, along with the other constellations, appears to turn right round the pole star in slightly less than 24 hours of mean time.

About the beginning of September the two "pointers" are on the meridian directly under the pole star at midnight. They make an angle of 45° with the vertical at 3 a.m., are at a right angle to it at 6 a.m., and are on the meridian above the pole

star at noon. Each day they pass these stations 4 minutes earlier, until about the beginning of March they are directly above the pole star at midnight and below it at noon.

Thus on any night a guess may be made at the time, if one knows the angle the "pointers" make

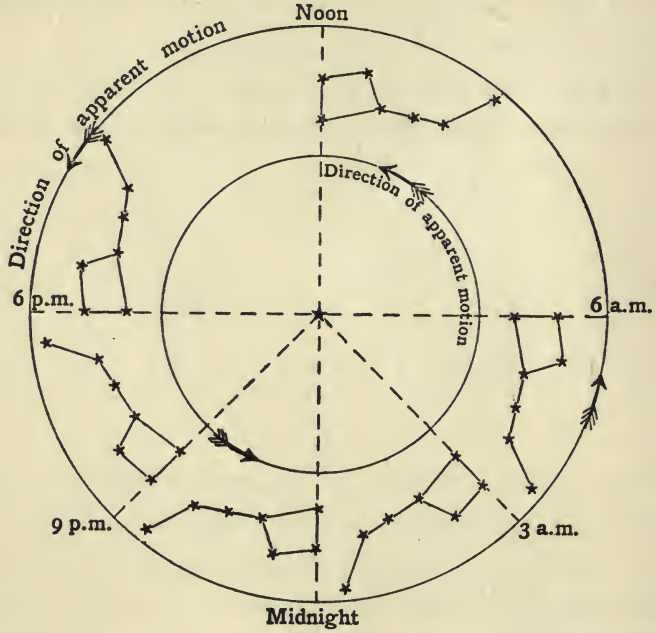

FIG. 84.
(About the beginning of September.)

with the vertical at midnight. It is a matter of mentally shifting round the figure above.

Another means of expressing time is furnished by the apparent movement of the stars. The period which elapses between two successive transits of the same star across the meridian of a place on the same side of the polar axis is constant, being the

same as the period of rotation of the earth round its axis. This period is called a *sidereal* day, which is divided into 24 hours, each hour of 60 minutes, and each minute of 60 seconds. Sidereal time differs slightly from mean time. The sidereal day is equal to 23 hours 56 minutes 4·1 seconds of mean time; and 24 hours of mean time is equal to 24 hours 3 minutes 56·6 seconds of sidereal time. So if a clock be regulated to show sidereal time, it will gain 3 minutes 56·6 seconds each day, or 24 hours in $365\frac{1}{4}$ days, in comparison with a clock that shows mean time. The advantage of using a sidereal clock, as is customary in observatories, is that by it every star crosses the meridian at the same time every day.

THE MOON.

The moon has an apparent motion in the heavens from West to East and takes about $29\frac{1}{2}$ days and 44 minutes to make a complete revolution round the earth. This period is called the *lunar* month. The crescent phase of the moon presents itself only while the line from the moon to the centre of the earth is at less than a right angle to the direction of the sun, and the convex edge of the crescent is then turned towards the sun.

The moon is only seen full when in opposition to the sun, and then rises about the time of sunset.

The moon rises and sets on an average about 48 minutes later each day; but this delay, due to its

orbital motion, varies considerably, as the "Harvest Moon," for instance, rises within 15 or 20 minutes of the previous day.

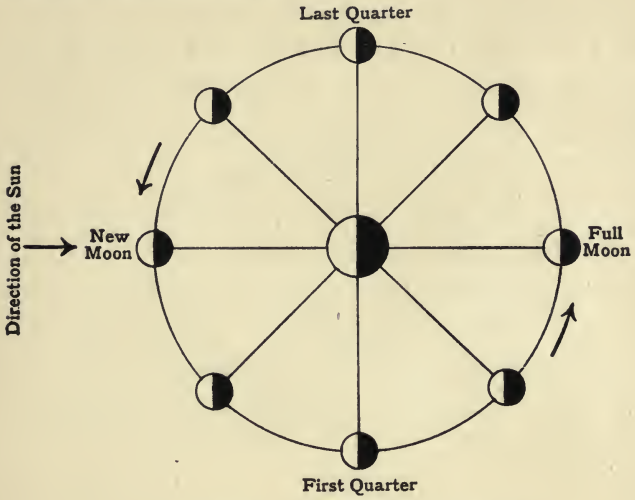

FIG. 85.

(The *phases* of the moon are shown above.)

The approximate positions of the moon at different hours are given below:

		At 6 P.M.	At Midnight.	At 6 A.M.
New Moon	(Invisible)			
First Quarter	(Forms a D)	South	West	
Full Moon		East	South	West
Last Quarter	(Forms a C)		East	South

FIG. 86.

They may also be represented symmetrically by a diagram :

FIG. 87.

The order of the quarters is reversed, and the quadrants of the compass are opposite the concavity of the crescents.

The cardinal points E., S. and W. can at least be subdivided, thus making the moon in the first quarter South-West at 9 p.m.

It is possible also that the inhabitants of a place may be able to tell one where the sun rises and sets there, even when they do not know the name of the village in which they live.* But in hilly country, at all events, the best system of *orientation* will not enable one to dispense with the services of an experienced guide, though it will serve to show whether he is to be trusted or not.

For if the man, the map, and the compass agree, there is little room for error—ever-present accompaniment of observation.

* The present writer once obtained some particularly unilluminating answers from certain children of the New Forest. "Where does that road lead?" he asked, indicating the highway to Southampton.

"Straight on!" replied the stolid natives.

"Well, what's this place anyhow?" he asked, impatiently, pointing to the double row of cottages that lined the roadside.

"Home!" they shouted in a tone of indignant protest at his stupidity.

Thus in broad daylight he was left completely in the dark.

THE USE OF THE MAP IN THE FIELD.

One is not able to employ the map with advantage until all its details have become sufficiently familiar to be recognised quickly and unmistakably.

Correct orientation being essential, one must first find on the map the position of the point one occupies and then place the lines of the map parallel to the corresponding lines on the ground.

In most cases, the point occupied is found at first sight, generally because its name or the name of a neighbouring point is given on the map. If, however, it has to be ascertained, one stretches a sheet of tracing paper on a block, or note-book, and through any point *O*, taken in the middle of this sheet, one sights three points in the landscape that are easily identifiable in the map (houses, steeples, factory chimneys, windmills, etc.). Having marked the directions *Oa*, *Ob*, *Oc* of these points, one places the tracing paper on the map and moves it until the lines *Oa*, *Ob*, *Oc* respectively pass through the graphic representations of the points *A*, *B*, and *C* in the landscape. A pin stuck through the point *O* will then mark the projection *o* of the point at which one is stationed.

In any case, when the starting point has been recognised or ascertained, one begins by glancing at the itinerary it is proposed to follow. A comparison is then made between the plan and the neighbouring country, the map being turned until each detail of the ground appears in the direction of an imaginary

line on the map that joins the starting point to the representation of this detail.

If one has a compass, one places the line 0 – 180 on a meridian of the map, with the zero uppermost. Then without touching the compass, one turns the map till the blue point of the needle (or arrow-head marked N) comes opposite the division which expresses the magnetic variation at the spot. The map is thus set to the true North, and one starts on the route without disturbing the orientation, making sure on the march from moment to moment that it has been maintained. One takes care, therefore, not to pass any detail in the landscape without having first recognised it on the map ; and equal care should be taken to identify each detail of the map with the corresponding detail on the ground.

In this way the observer will always be exactly informed about his position ; for whenever a mistake in direction is made he will be warned by the dissimilarity of the objects around him with the map. It must be borne in mind that the plan will scarcely ever be placed as one is accustomed to see it, when one consults the map without a definite object. The writing will be more or less reversed ; but, if in order to read a name the orientation of the map has to be altered, it must, without fail, be re-adjusted, so as to avoid mistakes.

Is it necessary to add that in this manner the positions of the points are similar in the map and in the landscape ? One finds to the right or the left

of one in the country the point which is placed on the map on the same side of the road followed.

It may happen that a detail of the map is not to be found in the country, or that a detail of the country is not given in the map. Although frequently revised, the General Staff map is never of the day. Details of planimetry are subject to constant change, new roads are made, houses built, etc. Moreover, certain details, necessarily grouped on the map because of the smallness of the scale, can be distinguished from each other in the country. There is no reason to be astonished, but it is as well to make sure by taking into account the details of elevation (*nivellement*) which do not vary.

On leaving a town of any importance, one should not fail to reconnoitre carefully the country side towards which one ought to march. Orientation is not easy in a maze of streets, and it is better to ask one's way than to run the risk of a mistake, which is always annoying, even when soon discovered.

On a fairly clear night, or if the use of a lamp is allowed, the map can render considerable assistance, although objects along the roadside are difficult to see. At any rate, information previously derived from the map and committed to memory will be of service on night marches.

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